Unit-2 DAA

CSE-4th sem

Brute force approach

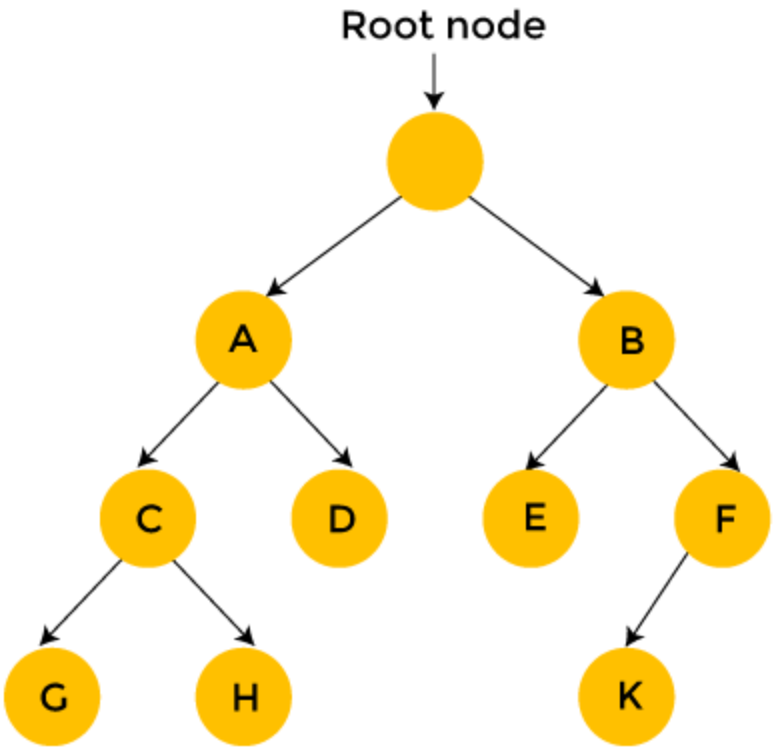
A brute force approach is an approach that finds all the possible solutions to find a satisfactory solution to a given problem. The brute force algorithm tries out all the possibilities till a satisfactory solution is not found.

Such an algorithm can be of two types:

- **Optimizing:** In this case, the best solution is found. To find the best solution, it may either find all the possible solutions to find the best solution or if the value of the best solution is known, it stops finding when the best solution is found. For example: Finding the best path for the travelling salesman problem. Here best path means that travelling all the cities and the cost of travelling should be minimum.
- **Satisfying:** It stops finding the solution as soon as the satisfactory solution is found. Or example, finding the travelling salesman path which is within 10% of optimal.
- Often Brute force algorithms require exponential time. Various heuristics and optimization can be used:
- **Heuristic:** A rule of thumb that helps you to decide which possibilities we should look at first.
- **Optimization:** A certain possibilities are eliminated without exploring all of them.

Let's understand the brute force search through an example.

Suppose we have converted the problem in the form of the tree shown as below:



Brute force search considers each and every state of a tree, and the state is represented in the form of a node. As far as the starting position is concerned, we have two choices, i.e., A state and B state. We can either generate state A or state B. In the case of B state, we have two states, i.e., state E and F.

In the case of brute force search, each state is considered one by one. As we can observe in the above tree that the brute force search takes 12 steps to find the solution.

On the other hand, backtracking, which uses Depth-First search, considers the below states only when the state provides a feasible solution. Consider the above tree, start from the root node, then move to node A and then node C. If node C does not provide the feasible solution, then there is no point in considering the states G and H. We backtrack from node C to node A. Then, we move from node A to node D. Since node D does not provide the feasible solution, we discard this state and backtrack from node D to node A.

We move to node B, then we move from node B to node E. We move from node E to node K; Since k is a solution, so it takes 10 steps to find the solution. In this way, we eliminate a greater number of states in a single iteration. Therefore, we can say that backtracking is faster and more efficient than the brute force approach.

Advantages of a brute-force algorithm

The following are the advantages of the brute-force algorithm:

- This algorithm finds all the possible solutions, and it also guarantees that it finds the correct solution to a problem.
- This type of algorithm is applicable to a wide range of domains.
- It is mainly used for solving simpler and small problems.
- It can be considered a comparison benchmark to solve a simple problem and does not require any particular domain knowledge.

Disadvantages of a brute-force algorithm

The following are the disadvantages of the brute-force algorithm:

- It is an inefficient algorithm as it requires solving each and every state.
- It is a very slow algorithm to find the correct solution as it solves each state without considering whether the solution is feasible or not.
- The brute force algorithm is neither constructive nor creative as compared to other algorithms.

Greedy Algorithm

The greedy method is one of the strategies like Divide and conquer used to solve the problems. This method is used for solving optimization problems. An optimization problem is a problem that demands either maximum or minimum results. Let's understand through some terms.

The Greedy method is the simplest and straightforward approach. It is not an algorithm, but it is a technique. The main function of this approach is that the decision is taken on the basis of the currently available information. Whatever the current information is present, the decision is made without worrying about the effect of the current decision in future.

This technique is basically used to determine the feasible solution that may or may not be optimal. The feasible solution is a subset that satisfies the given criteria. The optimal solution is the solution which is the best and the most favorable solution in the subset. In the case of feasible, if more than one solution satisfies the given criteria then those solutions will be considered as the feasible, whereas the optimal solution is the best solution among all the solutions.

Characteristics of Greedy method

The following are the characteristics of a greedy method:

- To construct the solution in an optimal way, this algorithm creates two sets where one set contains all the chosen items, and another set contains the rejected items.
- A Greedy algorithm makes good local choices in the hope that the solution should be either feasible or optimal.

Components of Greedy Algorithm

The components that can be used in the greedy algorithm are:

- **Candidate set:** A solution that is created from the set is known as a candidate set.
- **Selection function:** This function is used to choose the candidate or subset which can be added in the solution.
- **Feasibility function:** A function that is used to determine whether the candidate or subset can be used to contribute to the solution or not.
- **Objective function:** A function is used to assign the value to the solution or the partial solution.
- **Solution function:** This function is used to intimate whether the complete function has been reached or not.

Applications of Greedy Algorithm

- It is used in finding the shortest path.
- It is used to find the minimum spanning tree using the prim's algorithm or the Kruskal's algorithm.
- It is used in a job sequencing with a deadline.
- This algorithm is also used to solve the fractional knapsack problem.

Pseudo code of Greedy Algorithm

```
1.  Algorithm Greedy (a, n)
2.  {
3.     Solution : = 0;
4.    for i = 0 to n do
5.    {
6.        x: = select(a);
7.       if feasible(solution, x)
8.      {
9.          Solution: = union(solution , x)
10.    }
11.       return solution;
12.  } }
```

The above is the greedy algorithm. Initially, the solution is assigned with zero value. We pass the array and number of elements in the greedy algorithm. Inside the for loop, we select the element one by one and checks whether the solution is feasible or not. If the solution is feasible, then we perform the union.

Let's understand through an example.

Suppose there is a problem 'P'. I want to travel from A to B shown as below:

P : A → B

The problem is that we have to travel this journey from A to B. There are various solutions to go from A to B. We can go from A to B by **walk, car, bike, train, aeroplane**, etc. There is a constraint in the journey that we have to travel this journey within 12 hrs. If I go by train or aeroplane then only, I can cover this distance within 12 hrs. There are many solutions to this problem but there are only two solutions that satisfy the constraint.

If we say that we have to cover the journey at the minimum cost. This means that we have to travel this distance as minimum as possible, so this problem is known as a minimization problem. Till now, we have two feasible solutions, i.e., one by train and another one by air. Since travelling by train will lead to the minimum cost so it is an optimal solution. An optimal solution is also the feasible solution, but providing the best result so that solution is the optimal solution with the minimum cost. There would be only one optimal solution.

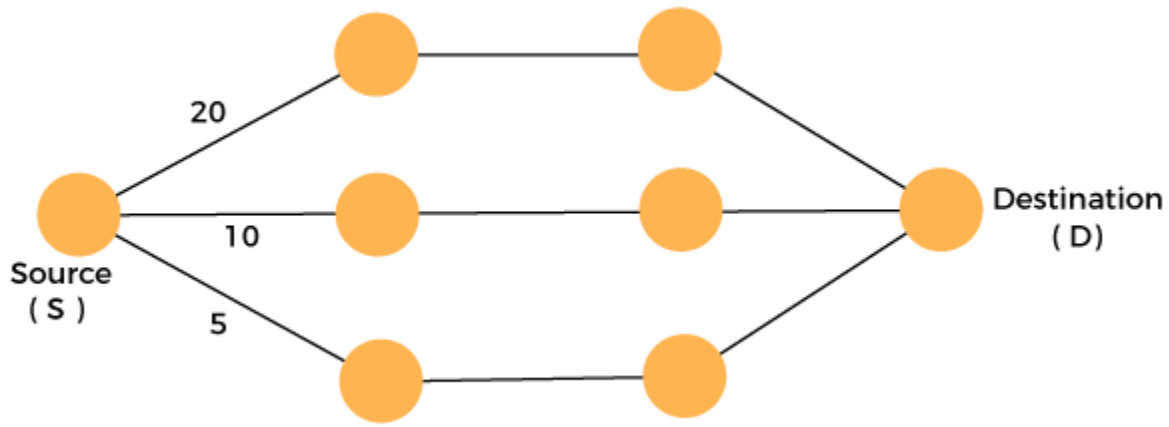
The problem that requires either minimum or maximum result then that problem is known as an optimization problem. Greedy method is one of the strategies used for solving the optimization problems.

Disadvantages of using Greedy algorithm

Greedy algorithm makes decisions based on the information available at each phase without considering the broader problem. So, there might be a possibility that the greedy solution does not give the best solution for every problem.

It follows the local optimum choice at each stage with a intend of finding the global optimum. Let's understand through an example.

Consider the graph which is given below:



We have to travel from the source to the destination at the minimum cost. Since we have three feasible solutions having cost paths as 10, 20, and 5. 5 is the minimum cost path so it is the optimal solution. This is the local optimum, and in this way, we find the local optimum at each stage in order to calculate the global optimal solution.

Dynamic Programming

Dynamic programming is a technique that breaks the problems into sub-problems, and saves the result for future purposes so that we do not need to compute the result again. The subproblems are optimized to optimize the overall solution is known as optimal substructure property. The main use of dynamic programming is to solve optimization problems. Here, optimization problems mean that when we are trying to find out the minimum or the maximum solution of a problem. The dynamic programming guarantees to find the optimal solution of a problem if the solution exists.

The definition of dynamic programming says that it is a technique for solving a complex problem by first breaking into a collection of simpler subproblems, solving each subproblem just once, and then storing their solutions to avoid repetitive computations.

Let's understand this approach through an example.

Consider an example of the Fibonacci series. The following series is the Fibonacci series:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ,…

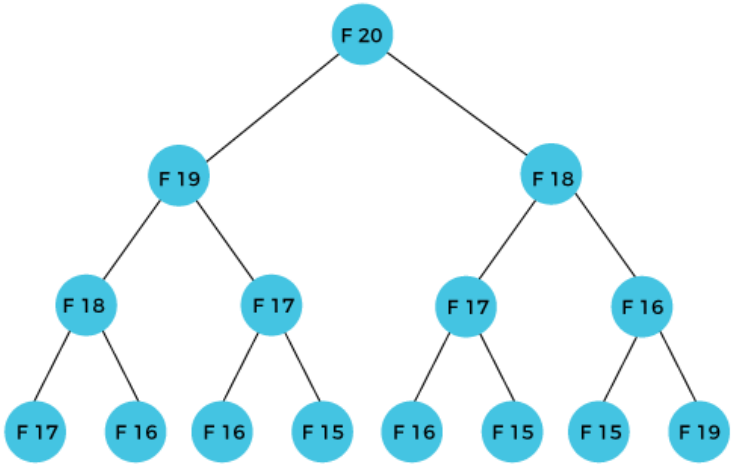
The numbers in the above series are not randomly calculated. Mathematically, we could write each of the terms using the below formula:

F(n) = F(n-1) + F(n-2),

With the base values F(0) = 0, and F(1) = 1. To calculate the other numbers, we follow the above relationship. For example, F(2) is the sum **f(0)** and **f(1),** which is equal to 1.

How can we calculate F(20)?

The F(20) term will be calculated using the nth formula of the Fibonacci series. The below figure shows that how F(20) is calculated.



As we can observe in the above figure that F(20) is calculated as the sum of F(19) and F(18). In the dynamic programming approach, we try to divide the problem into the similar subproblems. We are following this approach in the above case where F(20) into the similar subproblems, i.e., F(19) and F(18). If we recap the definition of dynamic programming that it says the similar subproblem should not be computed more than once. Still, in the above case, the subproblem is calculated twice. In the above example, F(18) is calculated two times; similarly, F(17) is also calculated twice. However, this technique is quite useful as it solves the similar subproblems, but we need to be cautious while storing the results because we are not particular about storing the result that we have computed once, then it can lead to a wastage of resources.

In the above example, if we calculate the F(18) in the right subtree, then it leads to the tremendous usage of resources and decreases the overall performance.

The solution to the above problem is to save the computed results in an array. First, we calculate F(16) and F(17) and save their values in an array. The F(18) is calculated by summing the values of F(17) and F(16), which are already saved in an array. The computed value of F(18) is saved in an array. The value of F(19) is calculated using the sum of F(18), and F(17), and their values are already saved in an array. The computed value of F(19) is stored in an array. The value of F(20) can be calculated by adding the values of

F(19) and F(18), and the values of both F(19) and F(18) are stored in an array. The final computed value of F(20) is stored in an array.

How does the dynamic programming approach work?

The following are the steps that the dynamic programming follows:

- It breaks down the complex problem into simpler subproblems.
- It finds the optimal solution to these sub-problems.
- It stores the results of subproblems (memoization). The process of storing the results of subproblems is known as memorization.
- It reuses them so that same sub-problem is calculated more than once.
- Finally, calculate the result of the complex problem.

The above five steps are the basic steps for dynamic programming. The dynamic programming is applicable that are having properties such as:

Those problems that are having overlapping sub problems and optimal sub structures. Here, optimal substructure means that the solution of optimization problems can be obtained by simply combining the optimal solution of all the subproblems.

In the case of dynamic programming, the space complexity would be increased as we are storing the intermediate results, but the time complexity would be decreased.

Approaches of dynamic programming

There are two approaches to dynamic programming:

- Top-down approach

- Bottom-up approach

Top-down approach

The top-down approach follows the memorization technique, while bottom-up approach follows the tabulation method. Here memorization is equal to the sum of recursion and caching. Recursion means calling the function itself, while caching means storing the intermediate results.

Advantages

- It is very easy to understand and implement.
- It solves the subproblems only when it is required.
- It is easy to debug.

Disadvantages

It uses the recursion technique that occupies more memory in the call stack. Sometimes when the recursion is too deep, the stack overflow condition will occur.

It occupies more memory that degrades the overall performance.

Let's understand dynamic programming through an example.

```
1. int fib(int n)
2. {
3.   if(n<0)
```

```
4.    error;
5.  if(n==0)
6.  return 0;
7.  if(n==1)
8. return 1;
9. sum = fib(n-1) + fib(n-2);
10.         }
```

In the above code, we have used the recursive approach to find out the Fibonacci series. When the value of 'n' increases, the function calls will also increase, and computations will also increase. In this case, the time complexity increases exponentially, and it becomes 2^n.

One solution to this problem is to use the dynamic programming approach. Rather than generating the recursive tree again and again, we can reuse the previously calculated value. If we use the dynamic programming approach, then the time complexity would be O(n).

When we apply the dynamic programming approach in the implementation of the Fibonacci series, then the code would look like:

```
1. static int count = 0;
2. int fib(int n)
3. {
4. if(memo[n]!= NULL)
5. return memo[n];
```

```
6.  count++;
7.    if(n<0)
8.    error;
9.   if(n==0)
10.          return 0;
11.          if(n==1)
12.         return 1;
13.         sum = fib(n-1) + fib(n-2);
14.         memo[n] = sum;
15.         }
```

In the above code, we have used the memorization technique in which we store the results in an array to reuse the values. This is also known as a top-down approach in which we move from the top and break the problem into sub-problems.

Bottom-Up approach

The bottom-up approach is also one of the techniques which can be used to implement the dynamic programming. It uses the tabulation technique to implement the dynamic programming approach. It solves the same kind of problems but it removes the recursion. If we remove the recursion, there is no stack overflow issue and no overhead of the recursive functions. In this tabulation technique, we solve the problems and store the results in a matrix.

There are two ways of applying dynamic programming:

- **Top-Down**
- **Bottom-Up**

The bottom-up is the approach used to avoid the recursion, thus saving the memory space. The bottom-up is an algorithm that starts from the beginning, whereas the recursive algorithm starts from the end and works backward. In the bottom-up approach, we start from the base case to find the answer for the end. As we know, the base cases in the Fibonacci series are 0 and 1. Since the bottom approach starts from the base cases, so we will start from 0 and 1.

Key points

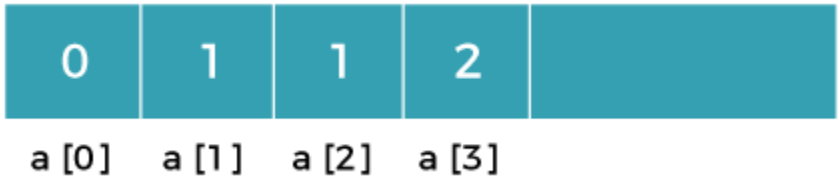
- We solve all the smaller sub-problems that will be needed to solve the larger sub-problems then move to the larger problems using smaller sub-problems.
- We use for loop to iterate over the sub-problems.
- The bottom-up approach is also known as the tabulation or table filling method.

Let's understand through an example.

Suppose we have an array that has 0 and 1 values at a[0] and a[1] positions, respectively shown as below:

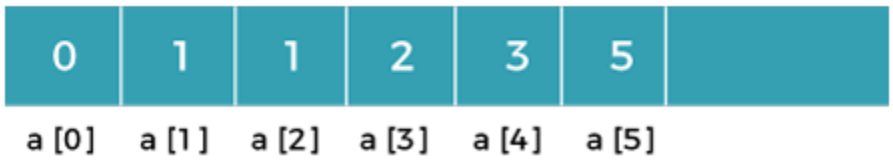
Since the bottom-up approach starts from the lower values, so the values at a[0] and a[1] are added to find the value of a[2] shown as below:

The value of a[3] will be calculated by adding a[1] and a[2], and it becomes 2 shown as below:



The value of a[4] will be calculated by adding a[2] and a[3], and it becomes 3 shown as below:

The value of a[5] will be calculated by adding the values of a[4] and a[3], and it becomes 5 shown as below:



The code for implementing the Fibonacci series using the bottom-up approach is given below:

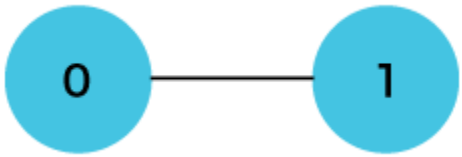
```
1. int fib(int n)
2. {
3.    int A[];
4.    A[0] = 0, A[1] = 1;
5.    for( i=2; i<=n; i++)
6.    {
7.        A[i] = A[i-1] + A[i-2]
```

```
8.      }
9.      return A[n];
10.         }
```

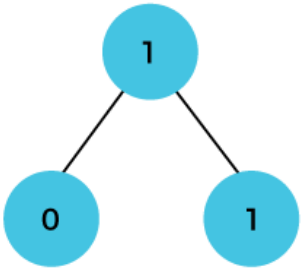
In the above code, base cases are 0 and 1 and then we have used for loop to find other values of Fibonacci series.

Let's understand through the diagrammatic representation.

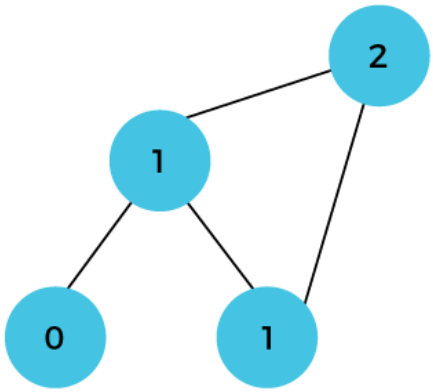
Initially, the first two values, i.e., 0 and 1 can be represented as:



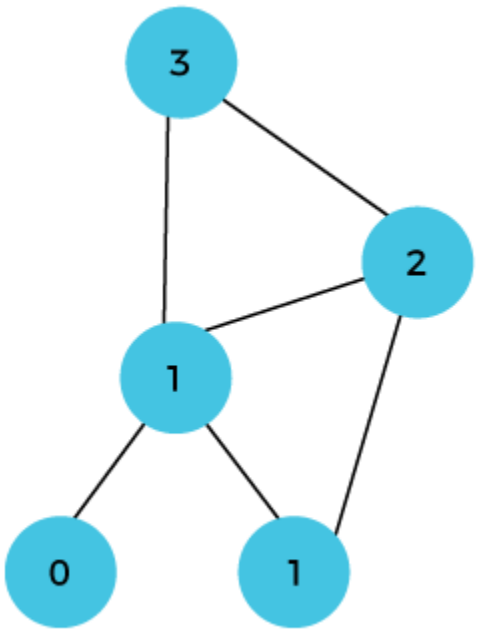
When i=2 then the values 0 and 1 are added shown as below:



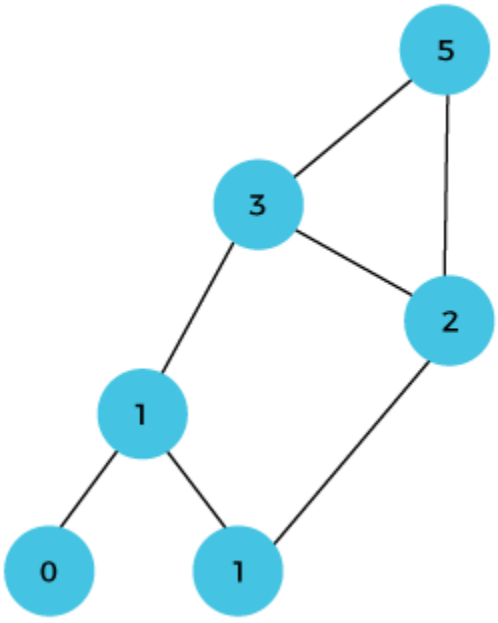
When i=3 then the values 1and 1 are added shown as below:



When i=4 then the values 2 and 1 are added shown as below:



When i=5, then the values 3 and 2 are added shown as below:



In the above case, we are starting from the bottom and reaching to the top.

Branch and bound

What is Branch and bound?

Branch and bound is one of the techniques used for problem solving. It is similar to the backtracking since it also uses the state space tree. It is used for solving the optimization problems and minimization problems. If we have given a maximization problem then we can convert it using the Branch and bound technique by simply converting the problem into a maximization problem.

Let's understand through an example.

Jobs = {j1, j2, j3, j4}

P = {10, 5, 8, 3}

d = {1, 2, 1, 2}

The above are jobs, problems and problems given. We can write the solutions in two ways which are given below:

Suppose we want to perform the jobs j1 and j2 then the solution can be represented in two ways:

The first way of representing the solutions is the subsets of jobs.

S1 = {j1, j4}

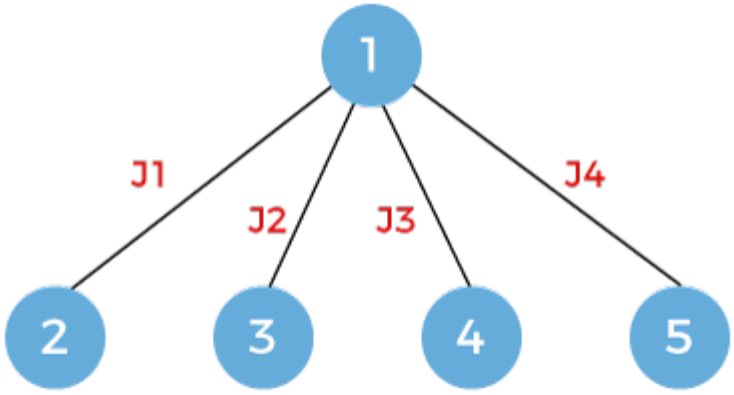
The second way of representing the solution is that first job is done, second and third jobs are not done, and fourth job is done.

S2 = {1, 0, 0, 1}

The solution s1 is the variable-size solution while the solution s2 is the fixed-size solution.

First, we will see the subset method where we will see the variable size.

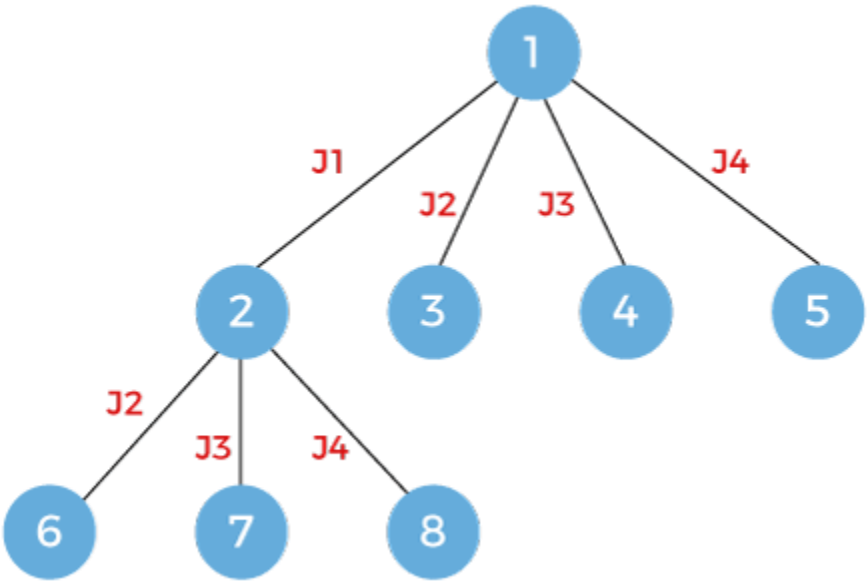
First method:



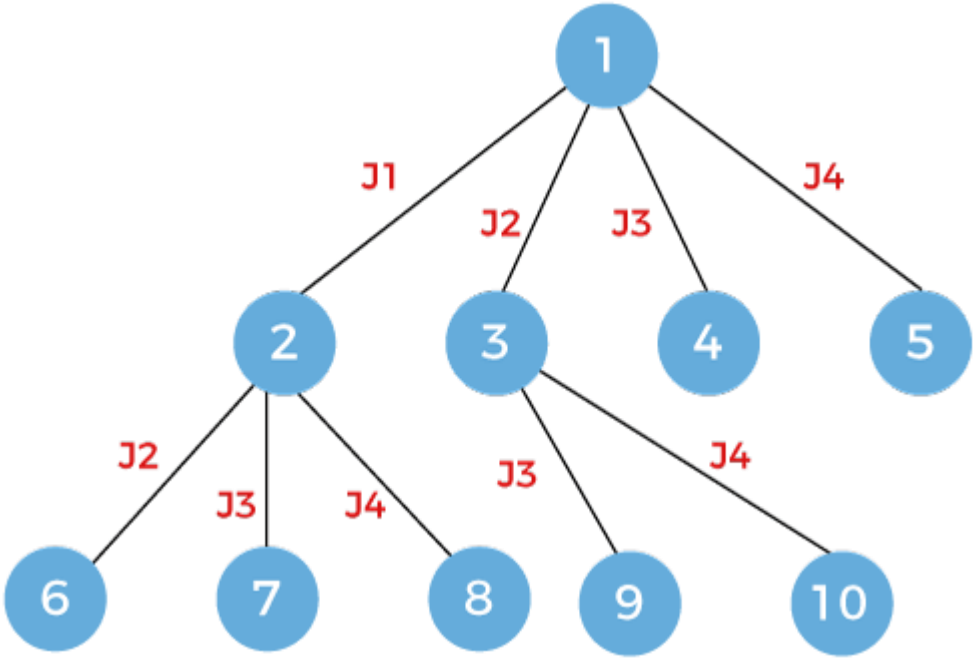
In this case, we first consider the first job, then second job, then third job and finally we consider the last job.

As we can observe in the above figure that the breadth first search is performed but not the depth first search. Here we move breadth wise for exploring the solutions. In backtracking, we go depth-wise whereas in branch and bound, we go breadth wise.

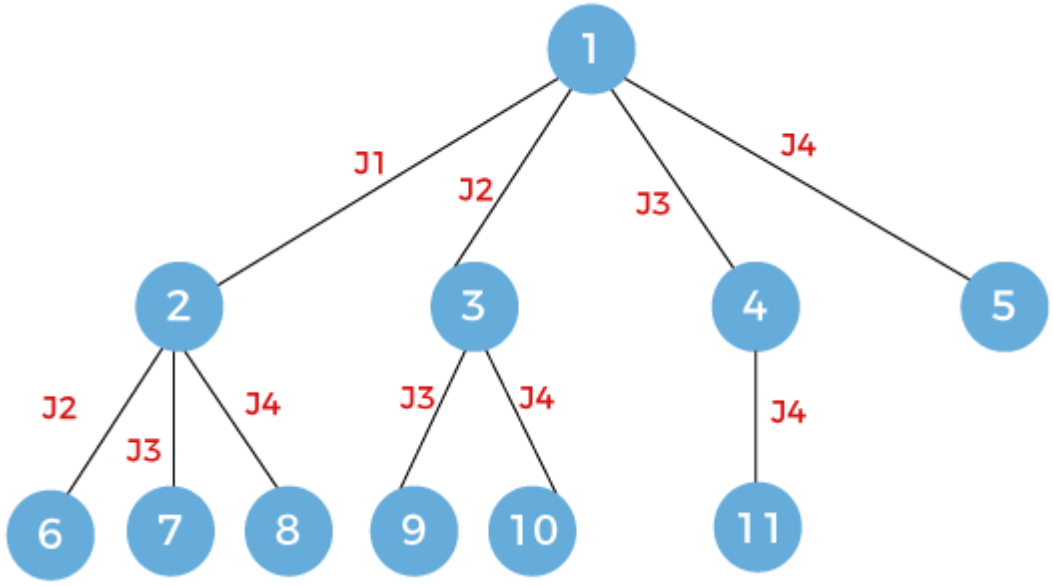
Now one level is completed. Once I take first job, then we can consider either j2, j3 or j4. If we follow the route then it says that we are doing jobs j1 and j4 so we will not consider jobs j2 and j3.



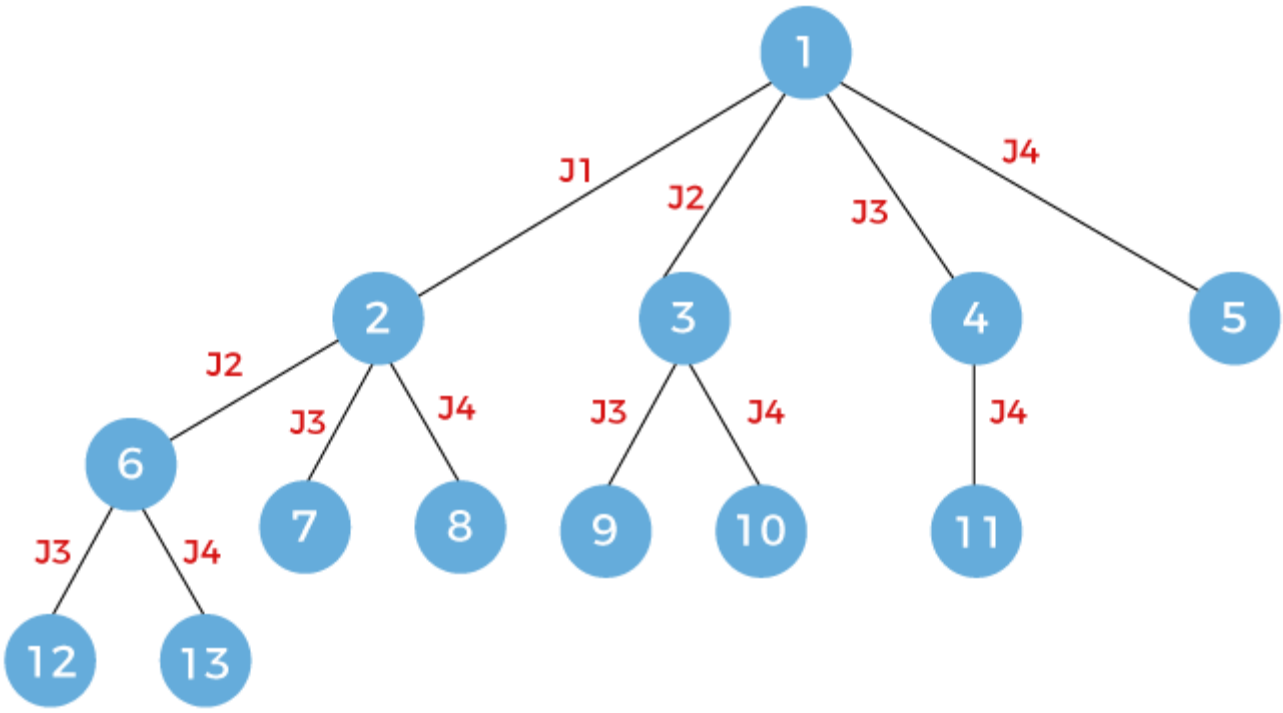
Now we will consider the node 3. In this case, we are doing job j2 so we can consider either job j3 or j4. Here, we have discarded the job j1.



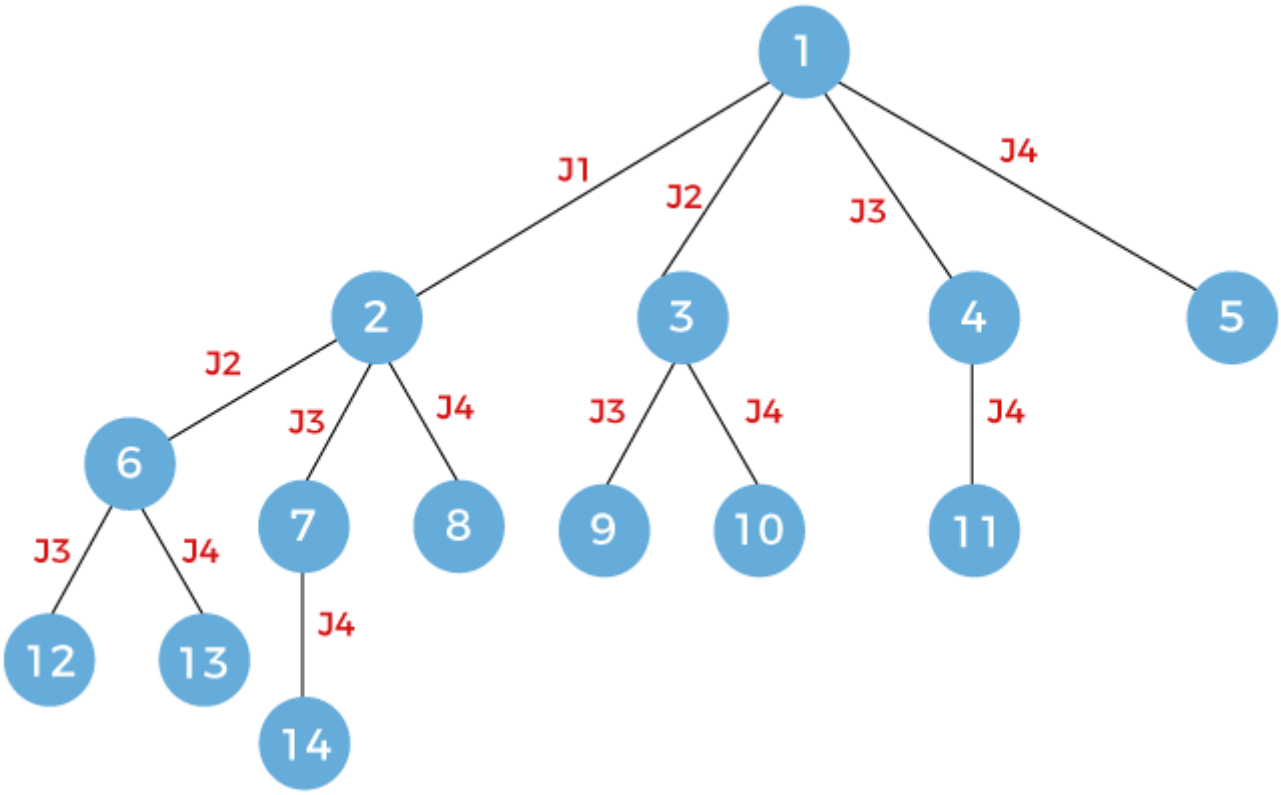
Now we will expand the node 4. Since here we are doing job j3 so we will consider only job j4.



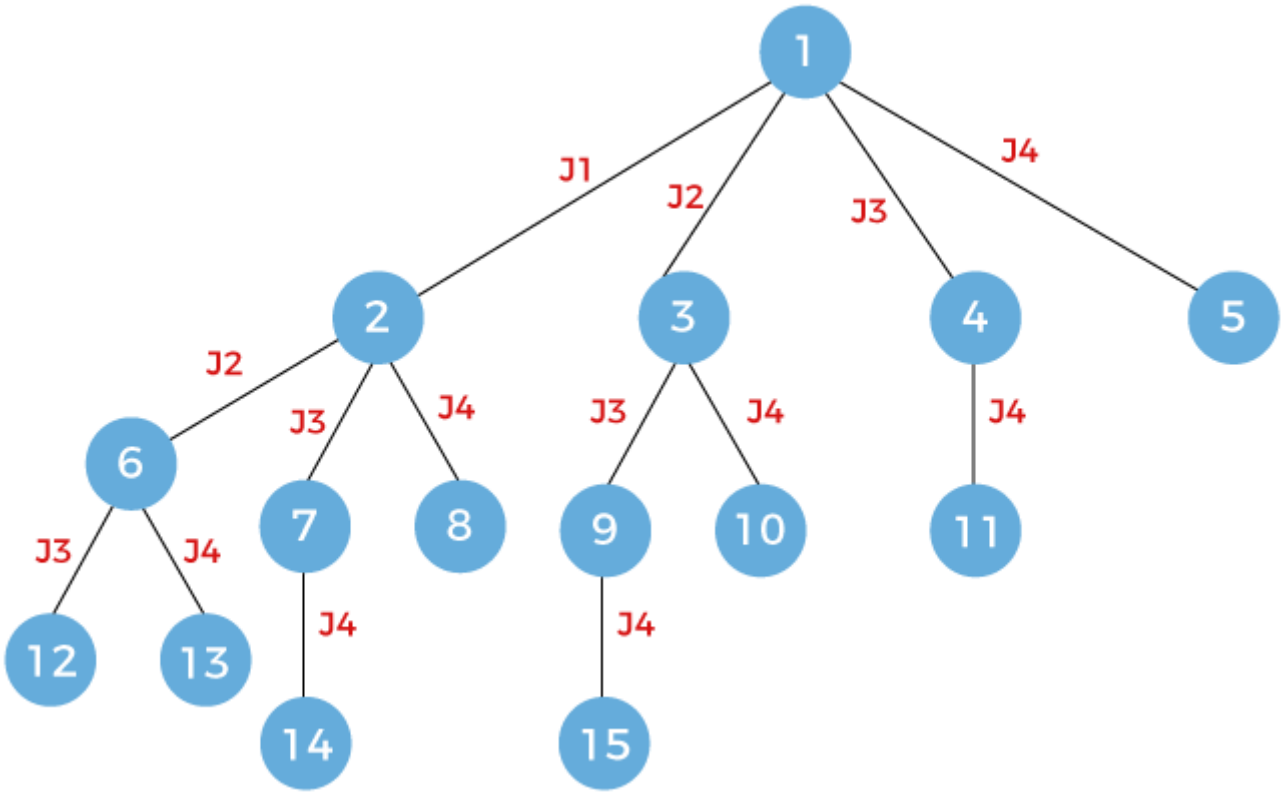
Now we will expand node 6, and here we will consider the jobs j3 and j4.



Now we will expand node 7 and here we will consider job j4.



Now we will expand node 9, and here we will consider job j4.



The last node, i.e., node 12 which is left to be expanded. Here, we consider job j4.

The above is the state space tree for the solution s1 = {j1, j4}

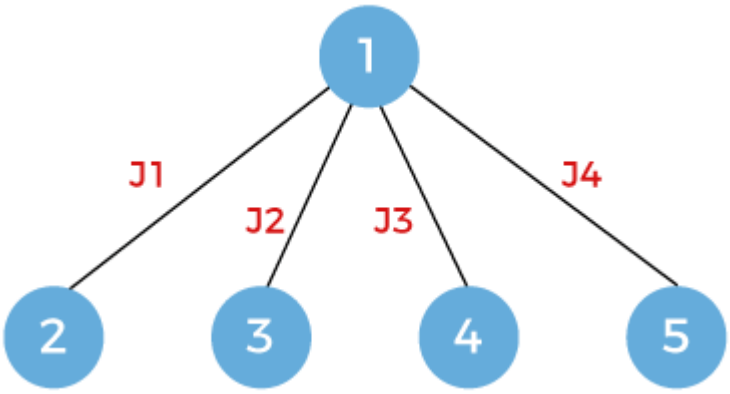
Second method:

We will see another way to solve the problem to achieve the solution s1.

First, we consider the node 1 shown as below:

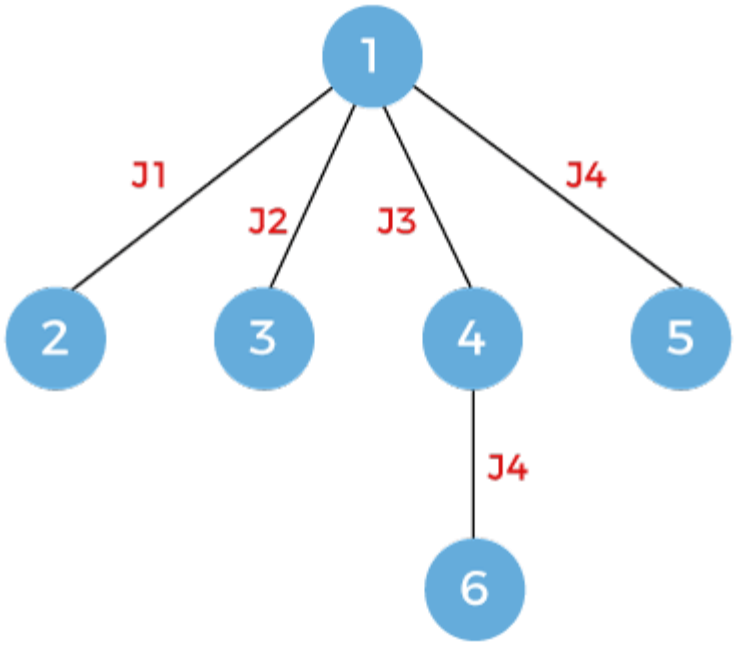
Now, we will expand the node 1. After expansion, the state space tree would be appeared as:

On each expansion, the node will be pushed into the stack shown as below:

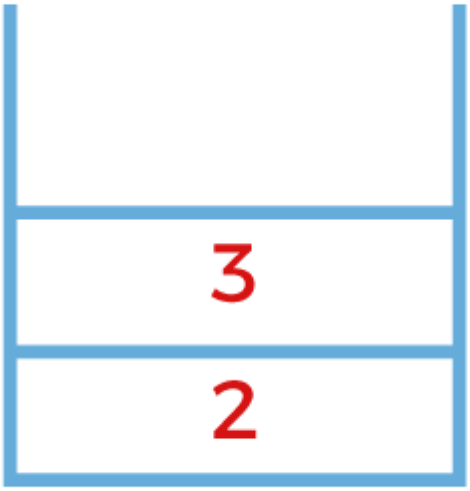


Now the expansion would be based on the node that appears on the top of the stack. Since the node 5 appears on the top of the stack, so we will expand the node 5. We will pop out the node 5 from the stack. Since the node 5 is in the last job, i.e., j4 so there is no further scope of expansion.

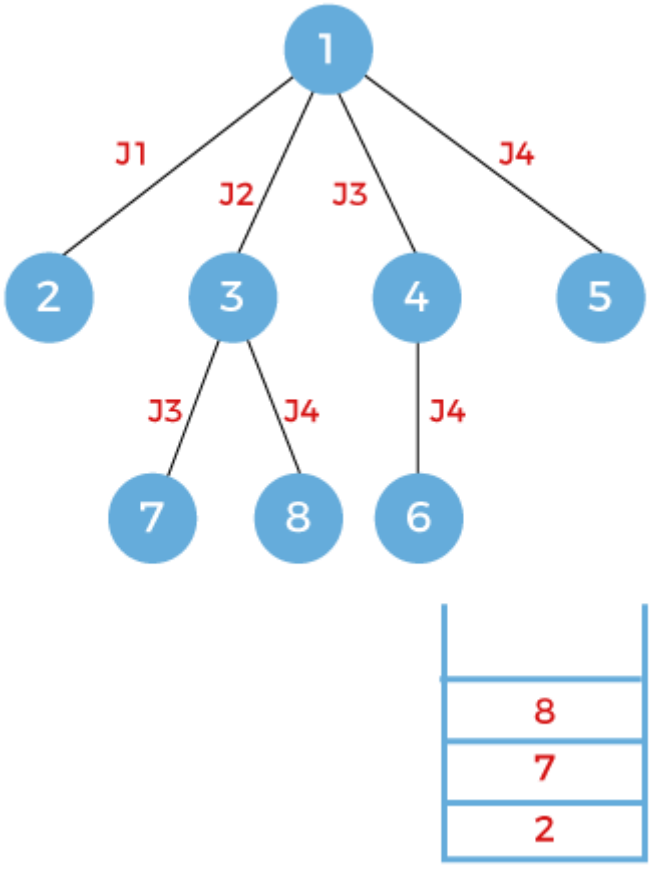
The next node that appears on the top of the stack is node 4. Pop out the node 4 and expand. On expansion, job j4 will be considered and node 6 will be added into the stack shown as below:



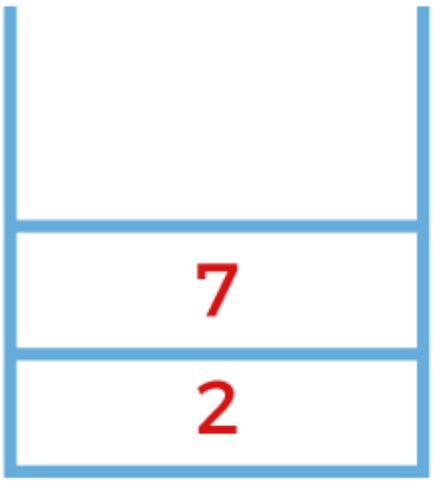
The next node is 6 which is to be expanded. Pop out the node 6 and expand. Since the node 6 is in the last job, i.e., j4 so there is no further scope of expansion.



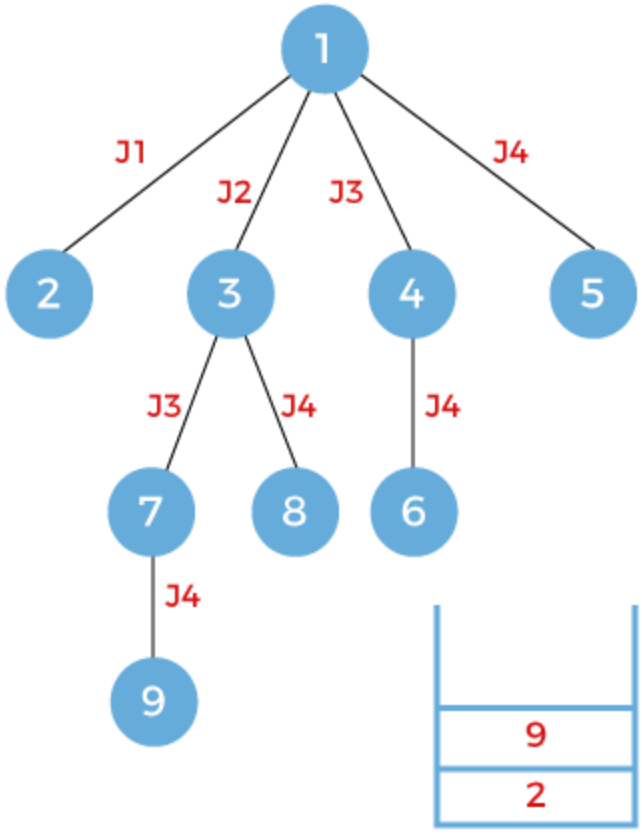
The next node to be expanded is node 3. Since the node 3 works on the job j2 so node 3 will be expanded to two nodes, i.e., 7 and 8 working on jobs 3 and 4 respectively. The nodes 7 and 8 will be pushed into the stack shown as below:



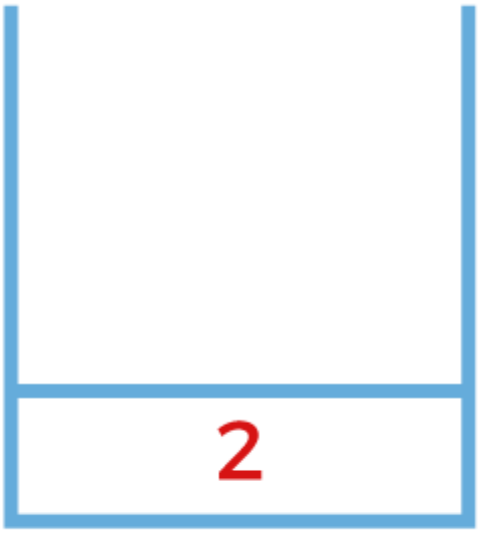
The next node that appears on the top of the stack is node 8. Pop out the node 8 and expand. Since the node 8 works on the job j4 so there is no further scope for the expansion.



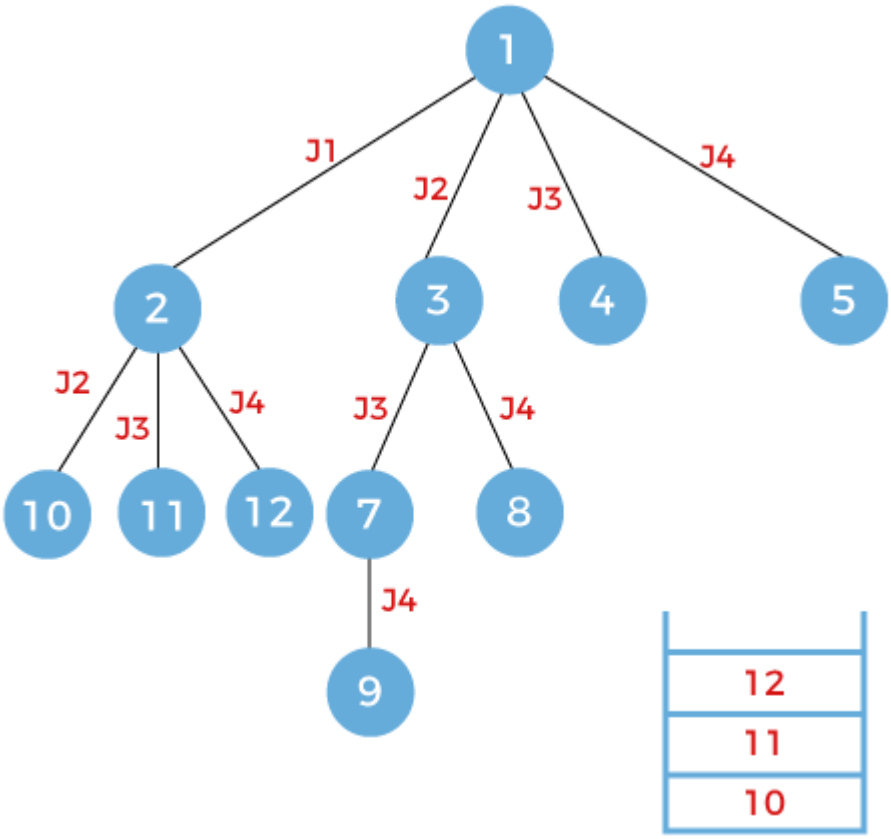
The next node that appears on the top of the stack is node 7. Pop out the node 7 and expand. Since the node 7 works on the job j3 so node 7 will be further expanded to node 9 that works on the job j4 as shown as below and the node 9 will be pushed into the stack.



The next node that appears on the top of the stack is node 9. Since the node 9 works on the job 4 so there is no further scope for the expansion.



The next node that appears on the top of the stack is node 2. Since the node 2 works on the job j1 so it means that the node 2 can be further expanded. It can be expanded upto three nodes named as 10, 11, 12 working on jobs j2, j3, and j4 respectively. There newly nodes will be pushed into the stack shown as below:



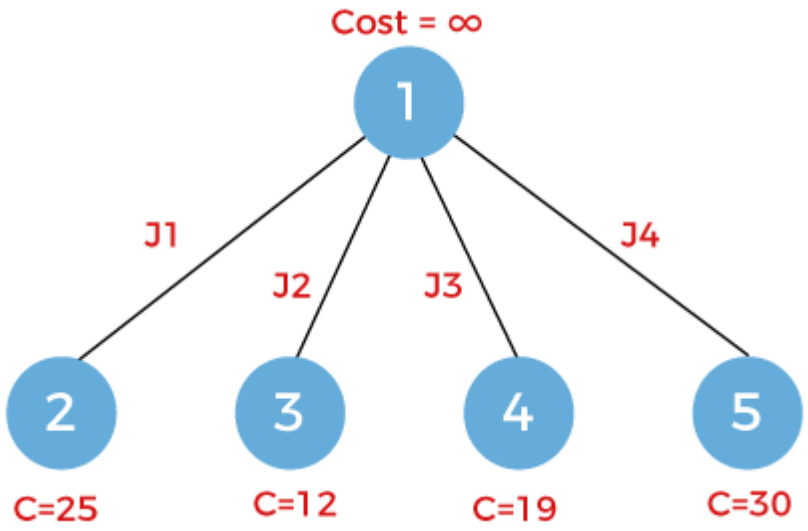
In the above method, we explored all the nodes using the stack that follows the LIFO principle.

Third method

There is one more method that can be used to find the solution and that method is Least cost branch and bound. In this technique, nodes are explored based on the cost of the node. The cost of the node can be defined using the problem and with the help of the given problem, we can define the cost function. Once the cost function is defined, we can define the cost of the node.

Let's first consider the node 1 having cost infinity shown as below:

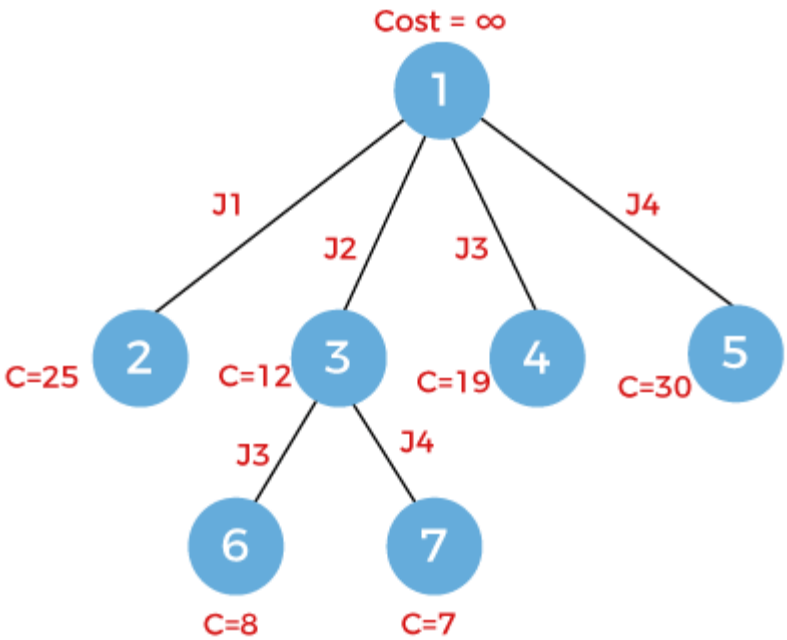
Now we will expand the node 1. The node 1 will be expanded into four nodes named as 2, 3, 4 and 5 shown as below:



Let's assume that cost of the nodes 2, 3, 4, and 5 are 25, 12, 19 and 30 respectively.

Since it is the least cost branch n bound, so we will explore the node which is having the least cost. In the above figure, we can observe that the node with a minimum cost is node 3. So, we will explore the node 3 having cost 12.

Since the node 3 works on the job j2 so it will be expanded into two nodes named as 6 and 7 shown as below:



The node 6 works on job j3 while the node 7 works on job j4. The cost of the node 6 is 8 and the cost of the node 7 is 7. Now we have to select the node which is having the minimum cost. The node 7 has the minimum cost so we will explore the node 7. Since the node 7 already works on the job j4 so there is no further scope for the expansion.

Backtracking

In this topic, we will learn about the backtracking, which is a very important skill set to solve recursive solutions. Recursive functions are those that calls itself more than once. Consider an example of Palindrome:

Initially, the function isPalindrome(S, 0, 8) is called once with the parameters isPalindrome(S, 1, 7). The recursive call isPalindrome(S, 1, 7) is called once with the parameters isPalindrome(S, 2, 6).

Backtracking is one of the techniques that can be used to solve the problem. We can write the algorithm using this strategy. It uses the Brute force search to solve the problem, and the brute force search says that for the given problem, we try to make all the possible solutions and pick out the best solution from all the desired solutions. This rule is also followed in dynamic programming, but dynamic programming is used for solving optimization problems. In contrast, backtracking is not used in solving optimization problems. Backtracking is used when we have multiple solutions, and we require all those solutions.

Backtracking name itself suggests that we are going back and coming forward; if it satisfies the condition, then return success, else we go back again. It is used to solve a problem in which a sequence of objects is chosen from a specified set so that the sequence satisfies some criteria.

When to use a Backtracking algorithm?

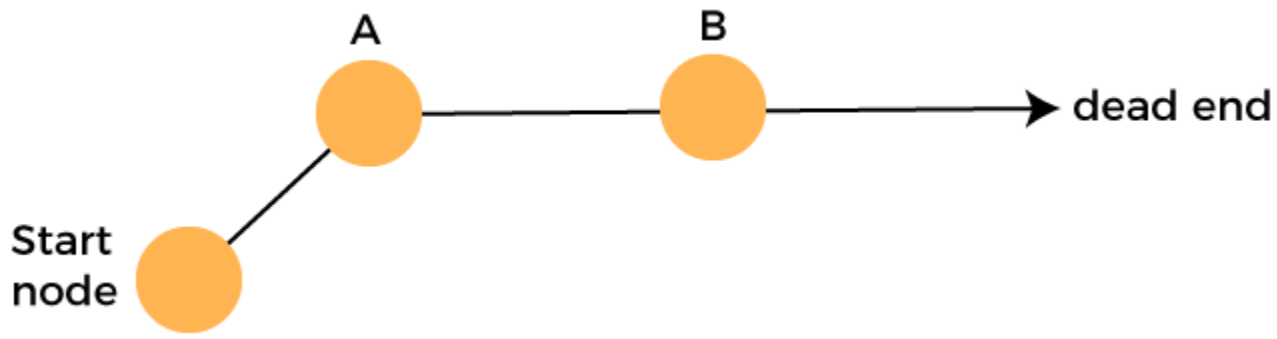
When we have multiple choices, then we make the decisions from the available choices. In the following cases, we need to use the backtracking algorithm:

- A piece of sufficient information is not available to make the best choice, so we use the backtracking strategy to try out all the possible solutions.
- Each decision leads to a new set of choices. Then again, we backtrack to make new decisions. In this case, we need to use the backtracking strategy.

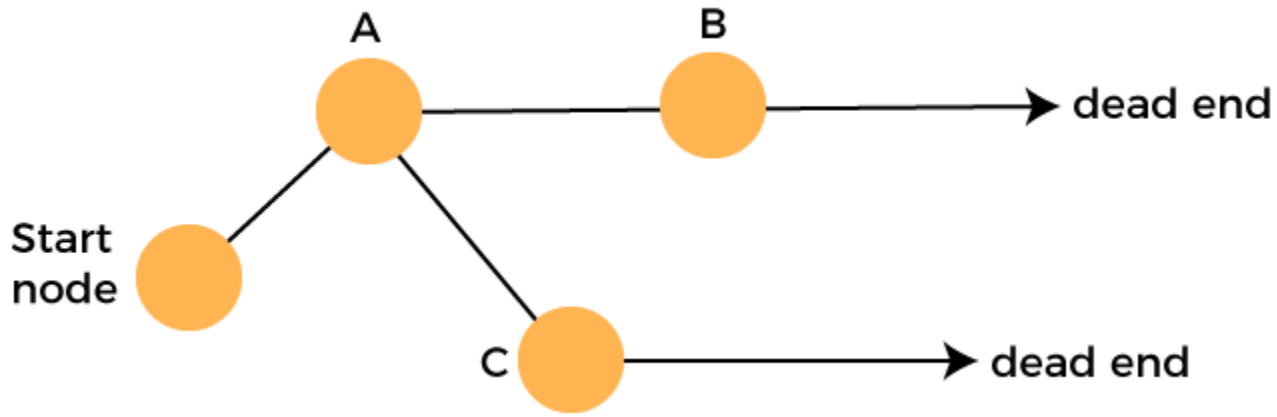
How does Backtracking work?

Backtracking is a systematic method of trying out various sequences of decisions until you find out that works. Let's understand through an example.

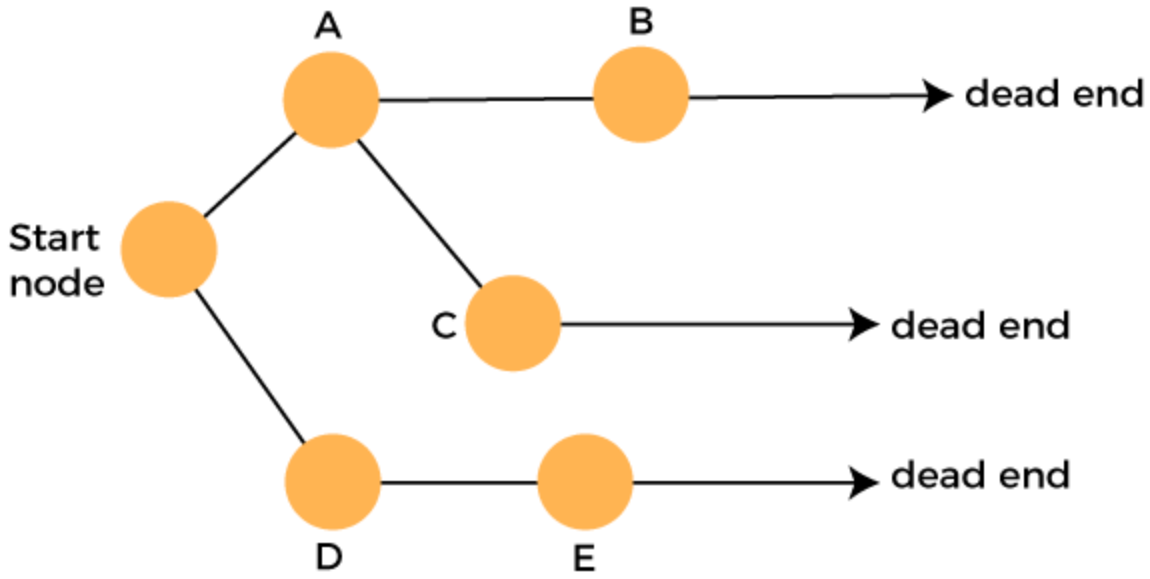
We start with a start node. First, we move to node A. Since it is not a feasible solution so we move to the next node, i.e., B. B is also not a feasible solution, and it is a dead-end so we backtrack from node B to node A.



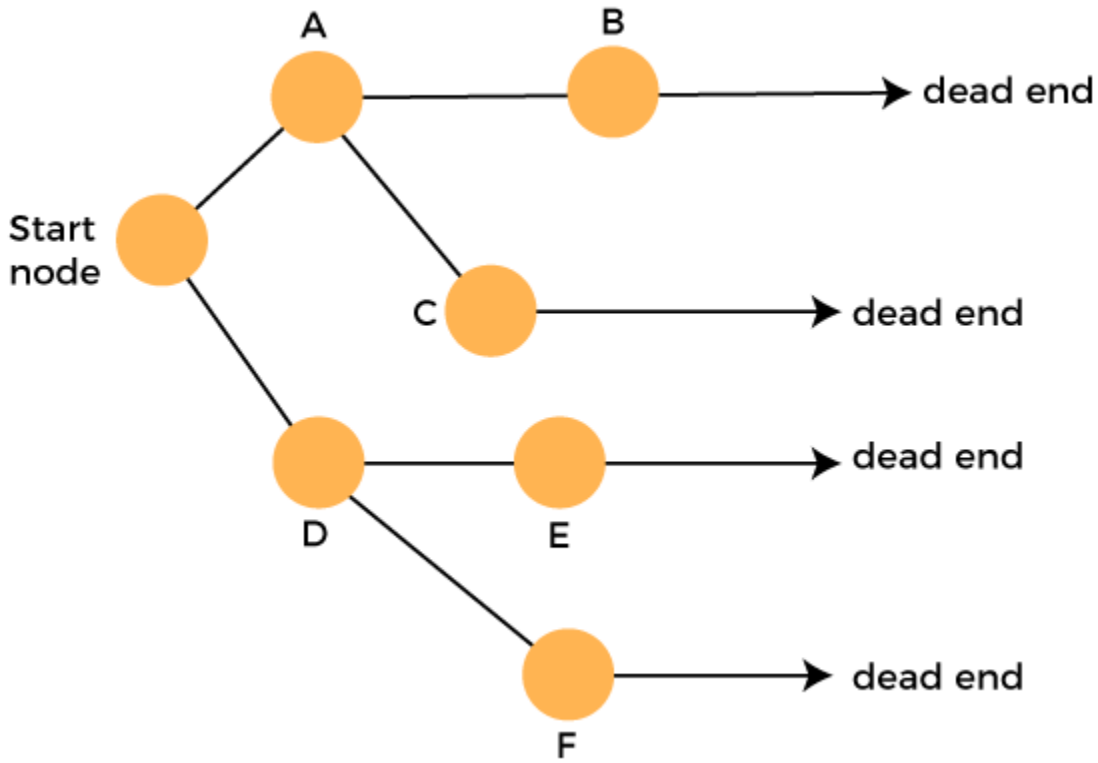
Suppose another path exists from node A to node C. So, we move from node A to node C. It is also a dead-end, so again backtrack from node C to node A. We move from node A to the starting node.



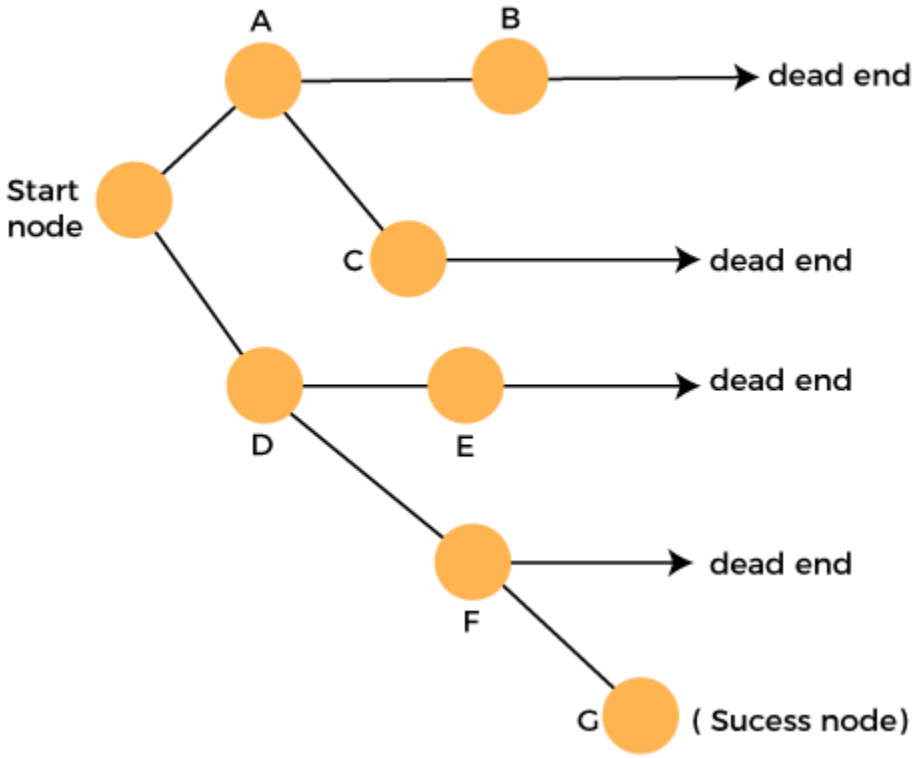
Now we will check any other path exists from the starting node. So, we move from start node to the node D. Since it is not a feasible solution so we move from node D to node E. The node E is also not a feasible solution. It is a dead end so we backtrack from node E to node D.



Suppose another path exists from node D to node F. So, we move from node D to node F. Since it is not a feasible solution and it's a dead-end, we check for another path from node F.



Suppose there is another path exists from the node F to node G so move from node F to node G. The node G is a success node.



The terms related to the backtracking are:

- **Live node:** The nodes that can be further generated are known as live nodes.
- **E node:** The nodes whose children are being generated and become a success node.
- **Success node:** The node is said to be a success node if it provides a feasible solution.
- **Dead node:** The node which cannot be further generated and also does not provide a feasible solution is known as a dead node.

Many problems can be solved by backtracking strategy, and that problems satisfy complex set of constraints, and these constraints are of two types:

- **Implicit constraint:** It is a rule in which how each element in a tuple is related.
- **Explicit constraint:** The rules that restrict each element to be chosen from the given set.

Applications of Backtracking

- N-queen problem

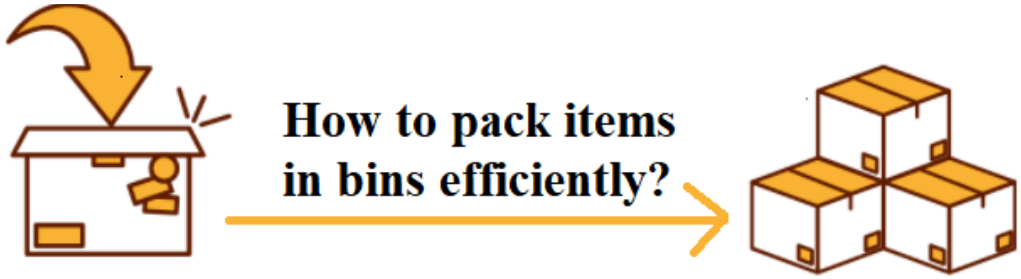
- Sum of subset problem
- Graph coloring
- Hamiliton cycle

Difference between the Backtracking and Recursion

Recursion is a technique that calls the same function again and again until you reach the base case. Backtracking is an algorithm that finds all the possible solutions and selects the desired solution from the given set of solutions.

Bin Packing problem

Bin Packing problem involves assigning n items of different weights and bins each of capacity c to a bin such that number of total used bins is minimized. It may be assumed that all items have weights smaller than bin capacity.



Let us get started with Bin Packing problem.

Mathematical Formulation of Bin Packing

The Bin-Packing Problem (BPP) can also be described,using the terminology of
knapsack problems, as follows. Given n items and n knapsacks (or bins), with

Wj = weight of item j,

cj = capacity of each bin

assign each item to one bin so that the total weight of the items in each bin does
not exceed c and the number of bins used is a minimum. A possible mathematical
formulation of the problem is

$$\text{minimize} \quad z = \sum_{i=1}^{n} y_i$$

$$\text{subject to} \quad \sum_{j=1}^{n} w_j x_{ij} \leq c y_i, \quad i \in N = \{1, \ldots, n\},$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j \in N,$$

$$y_i = 0 \text{ or } 1, \quad i \in N,$$

$$x_{ij} = 0 \text{ or } 1, \quad i \in N, j \in N,$$

where

$$y_i = \begin{cases} 1 & \text{if bin } i \text{ is used;} \\ 0 & \text{otherwise,} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{if item } j \text{ is assigned to bin } i; \\ 0 & \text{otherwise.} \end{cases}$$

We will suppose, as is usual, that the weights Wj are positive integers. Hence,
without loss of generality, we will also assume that c is a positive integer

Wj < c for j belonging to N.

As you can see, we have a broad rule of approximation and not an exact algorithm. Such algorithms are called NP problems. In fact Bin Packing Problem is a NP-hard problem

A brief outline of Approximate Algorithms

Many optimization problems exist for which there is no known polynomial time algorithm for its execution.

Approximation algorithms allow for getting a solution close to the (optimal) solution of an optimization problem in polynomial time.

An algorithm is an α-approximation algorithm for an optimized problem if:

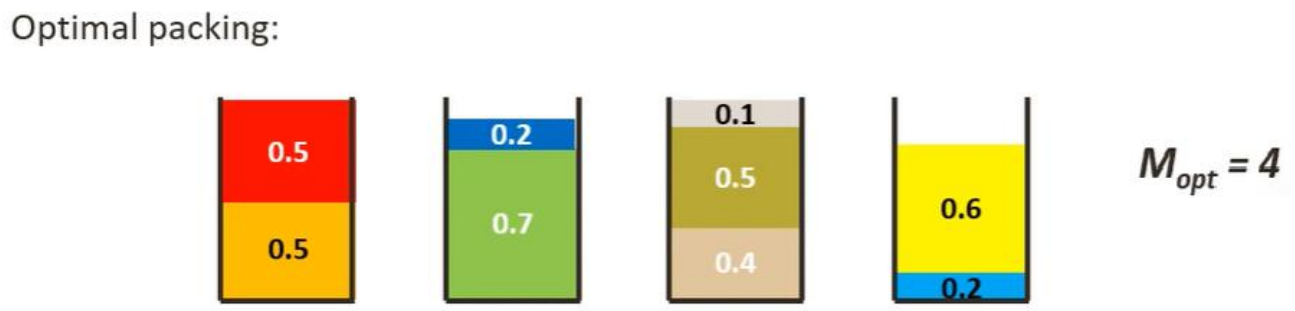
- The algorithm runs in polynomial time
- The algorithm always produces a solution that is within a factor of α of the optimal solution

For a given problem instance I,

Approximation ratio(α) = Algo(I)/z(I),

where Algo(I) is the algorithm under scrutiny and z(I) is the optimal solution.
For the Bin-Packing problem, let us consider bins of size 1

Assuming the sizes of the items be {0.5, 0.7, 0.5, 0.2, 0.4, 0.2, 0.5, 0.1, 0.6}.
The most optimal solution (z(I))for this instance I would be



Lower Bound on Bins

We can always find a lower bound on minimum number of bins required. The lower bound can be given as :

Min no. of bins >= Ceil ((Total Weight) / (Bin Capacity))

Let us now look at the various optimization algorithms for Bin Packing Problem.

Knapsack Problem

Given a set of items, each with a weight and a value, determine a subset of items to include in a collection so that the total weight is less than or equal to a given limit and the total value is as large as possible.

The knapsack problem is in combinatorial optimization problem. It appears as a subproblem in many, more complex mathematical models of real-world problems. One general approach to difficult problems is to identify the most restrictive constraint, ignore the others, solve a knapsack problem, and somehow adjust the solution to satisfy the ignored constraints.

Applications

In many cases of resource allocation along with some constraint, the problem can be derived in a similar way of Knapsack problem. Following is a set of example.

- Finding the least wasteful way to cut raw materials
- portfolio optimization
- Cutting stock problems

Problem Scenario

A thief is robbing a store and can carry a maximal weight of W into his knapsack. There are n items available in the store and weight of i^{th} item is w_i and its profit is p_i. What items should the thief take?

In this context, the items should be selected in such a way that the thief will carry those items for which he will gain maximum profit. Hence, the objective of the thief is to maximize the profit.

Based on the nature of the items, Knapsack problems are categorized as

- Fractional Knapsack
- Knapsack

Fractional Knapsack

In this case, items can be broken into smaller pieces, hence the thief can select fractions of items.

According to the problem statement,

- There are **n** items in the store
- Weight of i^{th} item $w_i > 0$
- Profit for i^{th} item $p_i > 0$ and
- Capacity of the Knapsack is **W**

In this version of Knapsack problem, items can be broken into smaller pieces. So, the thief may take only a fraction x_i of i^{th} item.

$$0 \leqslant x_i \leqslant 1$$

The i^{th} item contributes the weight $x_i.w_i$ to the total weight in the knapsack and profit $x_i.p_i$ to the total profit.

Hence, the objective of this algorithm is to

$$\text{maximize} \sum_{n=1}^{n} (x_i.p_i)$$

subject to constraint,

$$\sum_{n=1}^{n}(x_i.w_i) \leqslant W \sum_{n=1}^{n}(x_i.w_i) \leqslant W$$

It is clear that an optimal solution must fill the knapsack exactly, otherwise we could add a fraction of one of the remaining items and increase the overall profit.

Thus, an optimal solution can be obtained by

$$\sum_{n=1}^{n}(x_i.w_i) = W \sum_{n=1}^{n}(x_i.w_i) = W$$

In this context, first we need to sort those items according to the value of $p_i w_i$piwi, so that $p_{i+1} w_{i+1}$pi+1wi+1 ≤ $p_i w_i$piwi . Here, ***x*** is an array to store the fraction of items.

Algorithm: Greedy-Fractional-Knapsack (w[1..n], p[1..n], W)

```
for i = 1 to n
  do x[i] = 0
weight = 0
for i = 1 to n
  if weight + w[i] ≤ W then
    x[i] = 1
    weight = weight + w[i]
  else
    x[i] = (W - weight) / w[i]
    weight = W
    break
return x
```

Analysis

If the provided items are already sorted into a decreasing order of $p_i w_i$piwi, then the whileloop takes a time in ***O(n)***; Therefore, the total time including the sort is in ***O(n logn)***.

Example

Let us consider that the capacity of the knapsack ***W* = 60** and the list of provided items are shown in the following table −

Item	A	B	C	D
Profit	280	100	120	120
Weight	40	10	20	24

Ratio $(p_i w_i)$(piwi)	7	10	6	5

As the provided items are not sorted based on $p_i w_i$piwi. After sorting, the items are as shown in the following table.

Item	**B**	**A**	**C**	**D**
Profit	100	280	120	120
Weight	10	40	20	24
Ratio $(p_i w_i)$(piwi)	10	7	6	5

Solution

After sorting all the items according to $p_i w_i$piwi. First all of ***B*** is chosen as weight of ***B*** is less than the capacity of the knapsack. Next, item ***A*** is chosen, as the available capacity of the knapsack is greater than the weight of ***A***. Now, ***C*** is chosen as the next item. However, the whole item cannot be chosen as the remaining capacity of the knapsack is less than the weight of ***C***.

Hence, fraction of ***C*** (i.e. (60 − 50)/20) is chosen.

Now, the capacity of the Knapsack is equal to the selected items. Hence, no more item can be selected.

The total weight of the selected items is **10 + 40 + 20 * (10/20) = 60**

And the total profit is **100 + 280 + 120 * (10/20) = 380 + 60 = 440**

This is the optimal solution. We cannot gain more profit selecting any different combination of items.

0-1 Knapsack

0-1 Knapsack, items cannot be broken which means the thief should take the item as a whole or should leave it. This is reason behind calling it as 0-1 Knapsack.

Hence, in case of 0-1 Knapsack, the value of x_i can be either ***0*** or ***1***, where other constraints remain the same.

0-1 Knapsack cannot be solved by Greedy approach. Greedy approach does not ensure an optimal solution. In many instances, Greedy approach may give an optimal solution.

The following examples will establish our statement.

Example-1

Let us consider that the capacity of the knapsack is W = 25 and the items are as shown in the following table.

Item	**A**	**B**	**C**	**D**
Profit	24	18	18	10
Weight	24	10	10	7

Without considering the profit per unit weight ($\boldsymbol{p_i/w_i}$), if we apply Greedy approach to solve this problem, first item ***A*** will be selected as it will contribute max$_i$mum profit among all the elements.

After selecting item ***A***, no more item will be selected. Hence, for this given set of items total profit is ***24***. Whereas, the optimal solution can be achieved by selecting items, ***B*** and C, where the total profit is 18 + 18 = 36.

Example-2

Instead of selecting the items based on the overall benefit, in this example the items are selected based on ratio p_i/w_i. Let us consider that the capacity of the knapsack is W = 60 and the items are as shown in the following table.

Item	**A**	**B**	**C**

Price	100	280	120
Weight	10	40	20
Ratio	10	7	6

Using the Greedy approach, first item ***A*** is selected. Then, the next item ***B*** is chosen. Hence, the total profit is **100 + 280 = 380**. However, the optimal solution of this instance can be achieved by selecting items, ***B*** and ***C***, where the total profit is **280 + 120 = 400**.

Hence, it can be concluded that Greedy approach may not give an optimal solution.

To solve 0-1 Knapsack, Dynamic Programming approach is required.

Problem Statement

A thief is robbing a store and can carry a max$_i$mal weight of ***W*** into his knapsack. There are ***n*** items and weight of **ith** item is w_i and the profit of selecting this item is p_i. What items should the thief take?

Dynamic-Programming Approach

Let ***i*** be the highest-numbered item in an optimal solution **S** for **W** dollars. Then $S' = S - \{i\}$ is an optimal solution for $W - w_i$ dollars and the value to the solution ***S*** is V_i plus the value of the sub-problem.

We can express this fact in the following formula: define **c[i, w]** to be the solution for items **1,2, … , i** and the max$_i$mum weight **w**.

The algorithm takes the following inputs

- The max$_i$mum weight **W**
- The number of items **n**

- The two sequences **$v = <v_1, v_2, \ldots, v_n>$** and **$w = <w_1, w_2, \ldots, w_n>$**

Dynamic-0-1-knapsack (v, w, n, W)

```
for w = 0 to W do
  c[0, w] = 0
for i = 1 to n do
  c[i, 0] = 0
  for w = 1 to W do
    if w_i ≤ w then
      if v_i + c[i-1, w-w_i] then
        c[i, w] = v_i + c[i-1, w-w_i]
      else c[i, w] = c[i-1, w]
    else
      c[i, w] = c[i-1, w]
```

The set of items to take can be deduced from the table, starting at **c[n, w]** and tracing backwards where the optimal values came from.

If *c[i, w] = c[i-1, w]*, then item ***i*** is not part of the solution, and we continue tracing with **c[i-1, w]**. Otherwise, item ***i*** is part of the solution, and we continue tracing with **c[i-1, w-W]**.

Analysis

This algorithm takes $\theta(n, w)$ times as table c has $(n + 1).(w + 1)$ entries, where each entry requires $\theta(1)$ time to compute.

Job sequencing with deadline

Problem Statement

In job sequencing problem, the objective is to find a sequence of jobs, which is completed within their deadlines and gives maximum profit.

Solution

Let us consider, a set of ***n*** given jobs which are associated with deadlines and profit is earned, if a job is completed by its deadline. These jobs need to be ordered in such a way that there is maximum profit.

It may happen that all of the given jobs may not be completed within their deadlines.

Assume, deadline of **i**th job J_i is d_i and the profit received from this job is p_i. Hence, the optimal solution of this algorithm is a feasible solution with maximum profit.

Thus, D(i)>0D(i)>0 for 1⩽⩽n1⩽⩽n.

Initially, these jobs are ordered according to profit, i.e. p1⩾p2⩾p3⩾..⩾pnp1⩾p2⩾p3⩾..⩾pn.

Algorithm: Job-Sequencing-With-Deadline (D, J, n, k)

```
D(0) := J(0) := 0
k := 1
J(1) := 1   // means first job is selected
for i = 2 … n do
  r := k
  while D(J(r)) > D(i) and D(J(r)) ≠ r do
    r := r – 1
  if D(J(r)) ≤ D(i) and D(i) > r then
    for l = k … r + 1 by -1 do
      J(l + 1) := J(l)
      J(r + 1) := i
      k := k + 1
```

Analysis

In this algorithm, we are using two loops, one is within another. Hence, the complexity of this algorithm is O(n2)O(n2).

Example

Let us consider a set of given jobs as shown in the following table. We have to find a sequence of jobs, which will be completed within their deadlines and will give maximum profit. Each job is associated with a deadline and profit.

Job	J_1	J_2	J_3	J_4	J_5
Deadline	2	1	3	2	1
Profit	60	100	20	40	20

Solution

To solve this problem, the given jobs are sorted according to their profit in a descending order. Hence, after sorting, the jobs are ordered as shown in the following table.

Job	J_2	J_1	J_4	J_3	J_5
Deadline	1	2	2	3	1
Profit	100	60	40	20	20

From this set of jobs, first we select J_2, as it can be completed within its deadline and contributes maximum profit.

- Next, J_1 is selected as it gives more profit compared to J_4.
- In the next clock, J_4 cannot be selected as its deadline is over, hence J_3 is selected as it executes within its deadline.
- The job J_5 is discarded as it cannot be executed within its deadline.

Thus, the solution is the sequence of jobs (J_2, J_1, J_3), which are being executed within their deadline and gives maximum profit.

Total profit of this sequence is **100 + 60 + 20 = 180**.

Optimal Binary Search Tree

A Binary Search Tree (BST) is a tree where the key values are stored in the internal nodes. The external nodes are null nodes. The keys are ordered lexicographically, i.e. for each internal node all the keys in the left sub-tree are less than the keys in the node, and all the keys in the right sub-tree are greater.

When we know the frequency of searching each one of the keys, it is quite easy to compute the expected cost of accessing each node in the tree. An optimal binary search tree is a BST, which has minimal expected cost of locating each node

Search time of an element in a BST is ***O(n)***, whereas in a Balanced-BST search time is ***O(log n)***. Again the search time can be improved in Optimal Cost Binary Search Tree, placing the most frequently used data in the root and closer to the root element, while placing the least frequently used data near leaves and in leaves.

Here, the Optimal Binary Search Tree Algorithm is presented. First, we build a BST from a set of provided **n** number of distinct keys $< k_1, k_2, k_3, \ldots k_n >$. Here we assume, the probability of accessing a key K_i is p_i. Some dummy keys ($d_0, d_1, d_2, \ldots d_n$) are added as some searches may be performed for the values which are not present in the Key set K. We assume, for each dummy key d_i probability of access is q_i.

Optimal-Binary-Search-Tree(p, q, n)
e[1…n + 1, 0…n],
w[1…n + 1, 0…n],

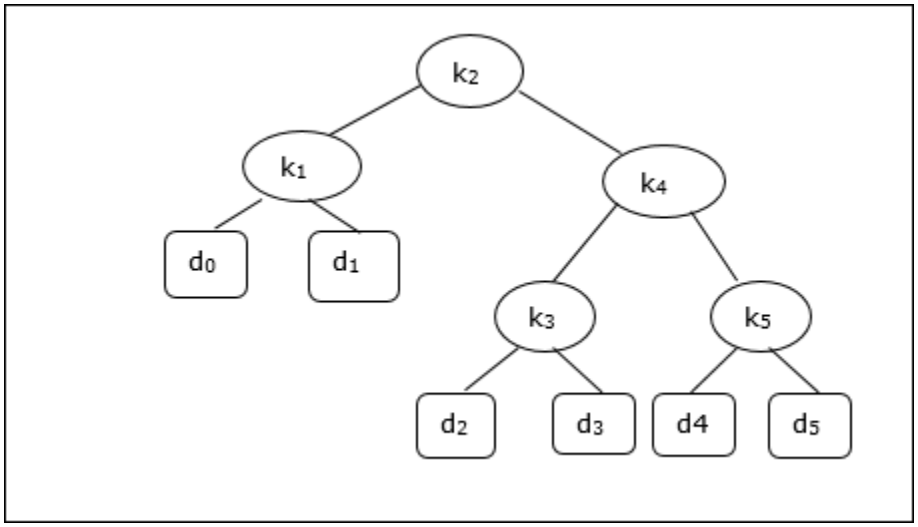
```
root[1…n + 1, 0…n]
for i = 1 to n + 1 do
  e[i, i - 1] := qi - 1
  w[i, i - 1] := qi - 1
for l = 1 to n do
  for i = 1 to n – l + 1 do
    j = i + l – 1 e[i, j] := ∞
    w[i, i] := w[i, i -1] + pi + qj
    for r = i to j do
      t := e[i, r - 1] + e[r + 1, j] + w[i, j]
      if t < e[i, j]
        e[i, j] := t
        root[i, j] := r
return e and root
```

Analysis

The algorithm requires **O (n^3)** time, since three nested **for** loops are used. Each of these loops takes on at most **n** values.

Example

Considering the following tree, the cost is 2.80, though this is not an optimal result.



Node	Depth	Probability	Contribution
k_1	1	0.15	0.30

k_2	0	0.10	0.10
k_3	2	0.05	0.15
k_4	1	0.10	0.20
k_5	2	0.20	0.60
d_0	2	0.05	0.15
d_1	2	0.10	0.30
d_2	3	0.05	0.20
d_3	3	0.05	0.20
d_4	3	0.05	0.20
d_5	3	0.10	0.40
Total			2.80

To get an optimal solution, using the algorithm discussed in this chapter, the following tables are generated.

In the following tables, column index is **i** and row index is **j**.

e	**1**	**2**	**3**	**4**	**5**	**6**
5	2.75	2.00	1.30	0.90	0.50	0.10

4	1.75	1.20	0.60	0.30	0.05	
3	1.25	0.70	0.25	0.05		
2	0.90	0.40	0.05			
1	0.45	0.10				
0	0.05					

w	**1**	**2**	**3**	**4**	**5**	**6**
5	1.00	0.80	0.60	0.50	0.35	0.10
4	0.70	0.50	0.30	0.20	0.05	
3	0.55	0.35	0.15	0.05		
2	0.45	0.25	0.05			
1	0.30	0.10				
0	0.05					

root	**1**	**2**	**3**	**4**	**5**
5	2	4	5	5	5

4	2	2	4	4	
3	2	2	3		
2	1	2			
1	1				

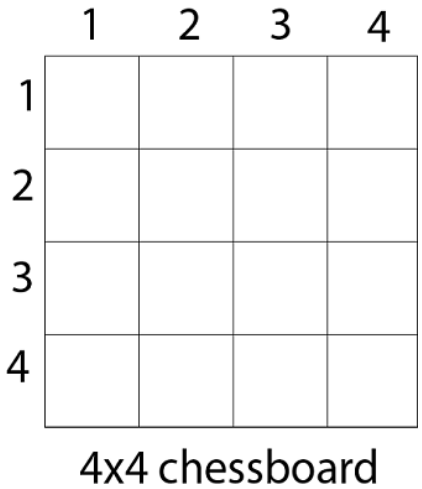
From these tables, the optimal tree can be formed.

N-Queens Problem

N - Queens problem is to place n - queens in such a manner on an n x n chessboard that no queens attack each other by being in the same row, column or diagonal.

It can be seen that for n =1, the problem has a trivial solution, and no solution exists for n =2 and n =3. So first we will consider the 4 queens problem and then generate it to n - queens problem.

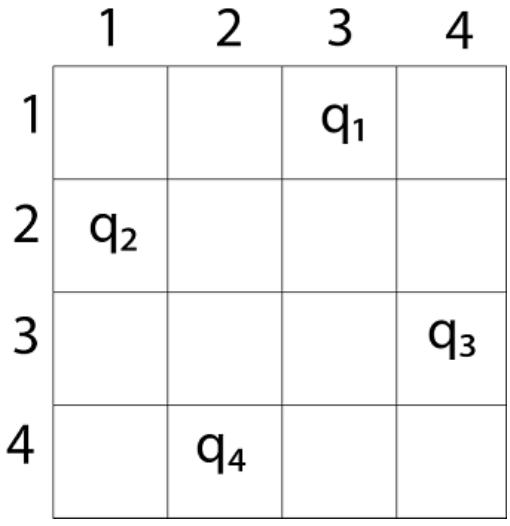
Given a 4 x 4 chessboard and number the rows and column of the chessboard 1 through 4.



4x4 chessboard

Since, we have to place 4 queens such as q_1 q_2 q_3 and q_4 on the chessboard, such that no two queens attack each other. In such a conditional each queen must be placed on a different row, i.e., we put queen "i" on row "i."

Now, we place queen q_1 in the very first acceptable position (1, 1). Next, we put queen q_2 so that both these queens do not attack each other. We find that if we place q_2 in column 1 and 2, then the dead end is encountered. Thus the first acceptable position for q_2 in column 3, i.e. (2, 3) but then no position is left for placing queen 'q_3' safely. So we backtrack one step and place the queen 'q_2' in (2, 4), the next best possible solution. Then we obtain the position for placing 'q_3' which is (3, 2). But later this position also leads to a dead end, and no place is found where 'q_4' can be placed safely. Then we have to backtrack till 'q_1' and place it to (1, 2) and then all other queens are placed safely by moving q_2 to (2, 4), q_3 to (3, 1) and q_4 to (4, 3). That is, we get the solution (2, 4, 1, 3). This is one possible solution for the 4-queens problem. For another possible solution, the whole method is repeated for all partial solutions. The other solutions for 4 - queens problems is (3, 1, 4, 2) i.e.



The implicit tree for 4 - queen problem for a solution (2, 4, 1, 3) is as follows:

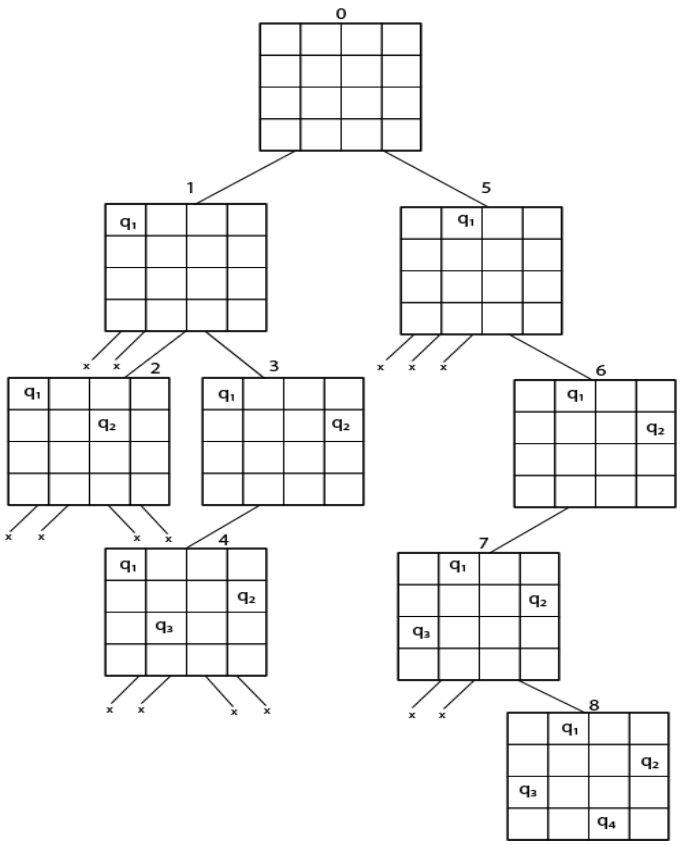
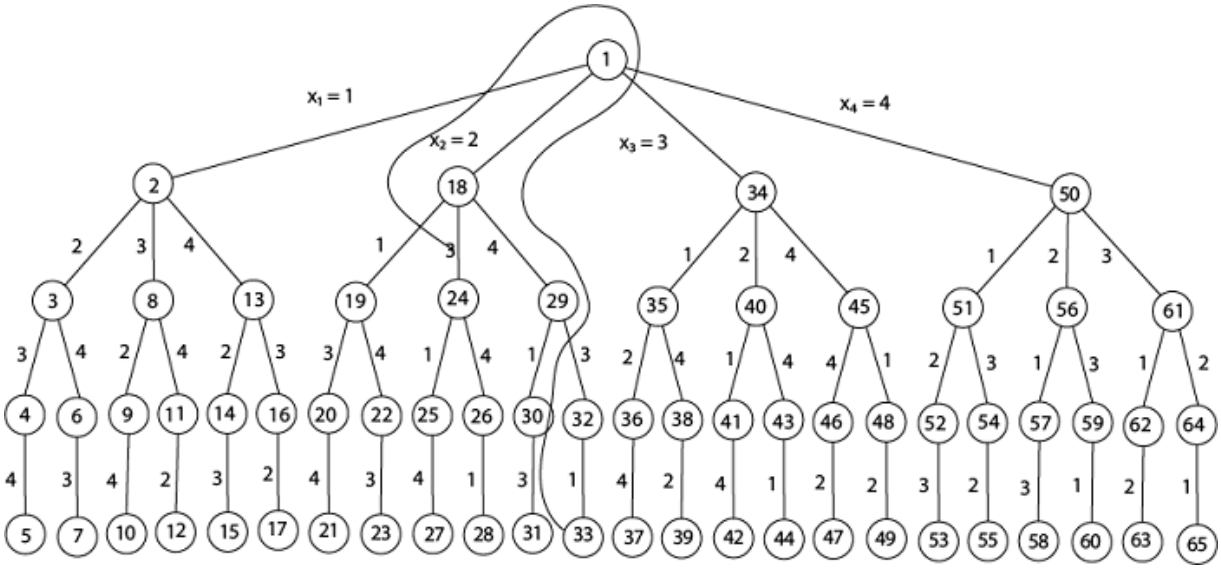


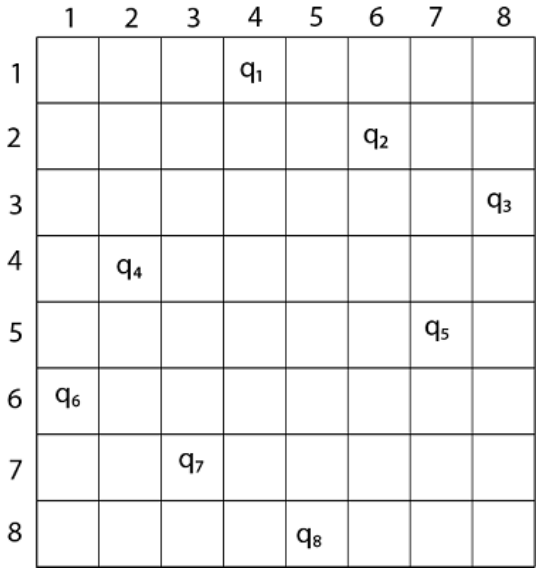
Fig shows the complete state space for 4 - queens problem. But we can use backtracking method to generate the necessary node and stop if the next node violates the rule, i.e., if two queens are attacking.



4 - Queens solution space with nodes numbered in DFS

It can be seen that all the solutions to the 4 queens problem can be represented as 4 - tuples (x_1, x_2, x_3, x_4) where x_i represents the column on which queen "q_i" is placed.

One possible solution for 8 queens problem is shown in fig:



1. Thus, the solution **for** 8 -queen problem **for** (4, 6, 8, 2, 7, 1, 3, 5).
2. If two queens are placed at position (i, j) and (k, l).
3. Then they are on same diagonal only **if** (i - j) = k - l or i + j = k + l.
4. The first equation implies that j - l = i - k.
5. The second equation implies that j - l = k - i.
6. Therefore, two queens lie on the duplicate diagonal **if** and only **if** |j-l|=|i-k|

Place (k, i) returns a Boolean value that is true if the kth queen can be placed in column i. It tests both whether i is distinct from all previous costs x_1, x_2,....x_{k-1} and whether there is no other queen on the same diagonal.

Using place, we give a precise solution to then n- queens problem.

1. Place (k, i)

```
2.   {
3.     For j ← 1 to k - 1
4.      do if (x [j] = i)
5.       or (Abs x [j]) - i) = (Abs (j - k))
6.    then return false;
7.     return true;
8.  }
```

Place (k, i) return true if a queen can be placed in the kth row and ith column otherwise return is false.

x [] is a global array whose final k - 1 values have been set. Abs (r) returns the absolute value of r.

```
1.  N - Queens (k, n)
2.  {
3.    For i ← 1 to n
4.       do if Place (k, i) then
5.    {
6.      x [k] ← i;
7.      if (k ==n) then
8.        write (x [1....n));
9.      else
10.              N - Queens (k + 1, n);
11.           }
12.        }
```

Huffman Codes

- (i) Data can be encoded efficiently using Huffman Codes.

- (ii) It is a widely used and beneficial technique for compressing data.
- (iii) Huffman's greedy algorithm uses a table of the frequencies of occurrences of each character to build up an optimal way of representing each character as a binary string.

Suppose we have 10^5 characters in a data file. Normal Storage: 8 bits per character (ASCII) - 8 x 10^5 bits in a file. But we want to compress the file and save it compactly. Suppose only six characters appear in the file:

	a	b	c	d	e	f	Total
Frequency	45	13	12	16	9	5	100

How can we represent the data in a Compact way?

(i) Fixed length Code: Each letter represented by an equal number of bits. With a fixed length code, at least 3 bits per character:

For example:

a	000
b	001
c	010
d	011
e	100
f	101

For a file with 10^5 characters, we need 3 x 10^5 bits.

(ii) A variable-length code: It can do considerably better than a fixed-length code, by giving many characters short code words and infrequent character long codewords.

For example:

a	0
b	101
c	100

d 111

e 1101

f 1100

Number of bits = (45 x 1 + 13 x 3 + 12 x 3 + 16 x 3 + 9 x 4 + 5 x 4) x 1000

= 2.24 x 10^5bits

Thus, 224,000 bits to represent the file, a saving of approximately 25%.This is an optimal character code for this file.

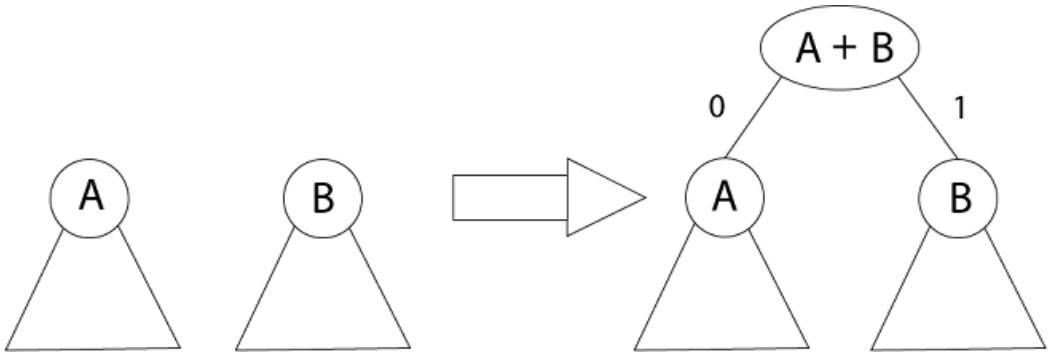
Prefix Codes:

The prefixes of an encoding of one character must not be equal to complete encoding of another character, e.g., 1100 and 11001 are not valid codes because 1100 is a prefix of some other code word is called prefix codes.

Prefix codes are desirable because they clarify encoding and decoding. Encoding is always simple for any binary character code; we concatenate the code words describing each character of the file. Decoding is also quite comfortable with a prefix code. Since no codeword is a prefix of any other, the codeword that starts with an encoded data is unambiguous.

Greedy Algorithm for constructing a Huffman Code:

Huffman invented a greedy algorithm that creates an optimal prefix code called a Huffman Code.



The algorithm builds the tree T analogous to the optimal code in a bottom-up manner. It starts with a set of |C| leaves (C is the number of characters) and performs |C| - 1 'merging' operations to create the final tree. In the Huffman algorithm 'n' denotes the quantity of a set of characters, z indicates the parent node, and x & y are the left & right child of z respectively.

Algorithm of Huffman Code

Huffman (C)

1. n=|C|
2. Q ← C
3. for i=1 to n-1
4. do
5. z= allocate-Node ()
6. x= left[z]=Extract-Min(Q)
7. y= right[z] =Extract-Min(Q)
8. f [z]=f[x]+f[y]
9. Insert (Q, z)
10. return Extract-Min (Q)

Example: Find an optimal Huffman Code for the following set of frequencies:

1. a: 50 b: 25 c: 15 d: 40 e: 75

Solution:

Given that: C = {a, b, c, d, e}

f (C) = {50, 25, 15, 40, 75}

n = 5

Q ⟵ c

i.e.

c	15	b	25	d	40	a	50	e	75

for i ⟵ 1 to 4

i = 1 Z ⟵ Allocate node

x ⟵ Extract-Min (Q)

y ⟵ Extract-Min (Q)

c	15	b	25	d	40	a	50	e	75

Left [z] ⟵ x

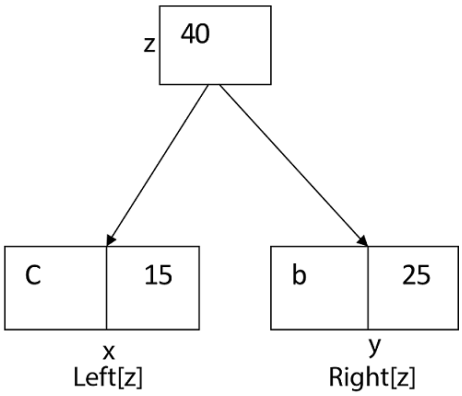
Right [z] ⟵ y

f (z) ⟵ f (x) + f (y) = 15 + 25

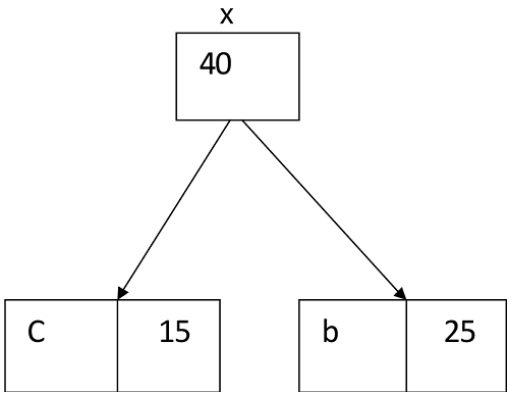
f (z) = 40

d	40	a	50	e	75

i.e.



Again for i=2



d	40

a	50

e	75

z ← Allocate node

x ← 40

y ← 40

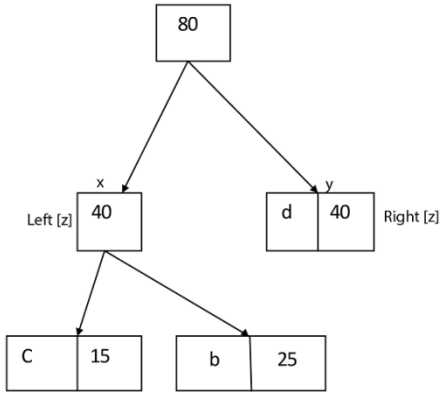
left [z] ← x

right [z] ← y

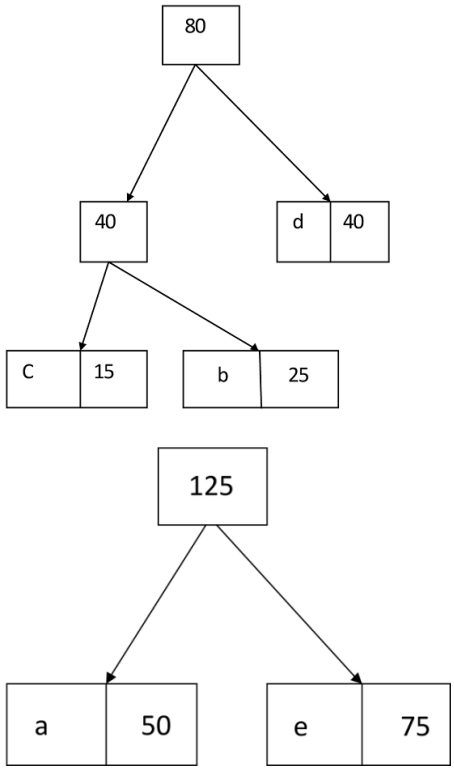
f (z) = 40 + 40 = 80

a	50

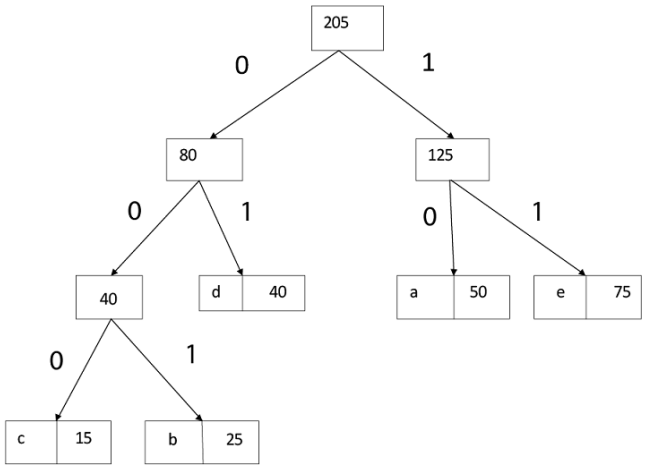
e	75



Similarly, we apply the same process we get



Thus, the final output is:



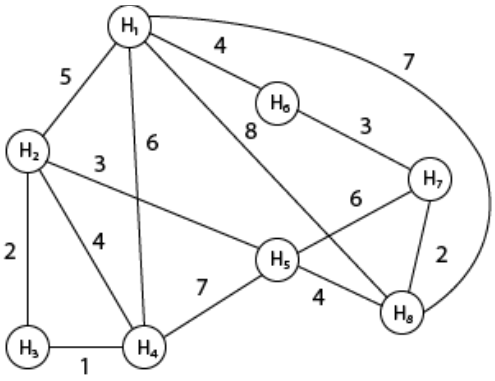
Travelling Sales Person Problem

The traveling salesman problems abide by a salesman and a set of cities. The salesman has to visit every one of the cities starting from a certain one (e.g., the hometown) and to return to the same city. The challenge of the problem is that the traveling salesman needs to minimize the total length of the trip.

Suppose the cities are x_1 x_2..... x_n where cost c_{ij} denotes the cost of travelling from city x_i to x_j. The travelling salesperson problem is to find a route starting and ending at x_1 that will take in all cities with the minimum cost.

Example: A newspaper agent daily drops the newspaper to the area assigned in such a manner that he has to cover all the houses in the respective area with minimum travel cost. Compute the minimum travel cost.

The area assigned to the agent where he has to drop the newspaper is shown in fig:



Solution: The cost- adjacency matrix of graph G is as follows:

$cost_{ij}$ =

	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8
H_1	0	5	0	6	0	4	0	7
H_2	5	0	2	4	3	0	0	0
H_3	0	2	0	1	0	0	0	0
H_4	6	4	1	0	7	0	0	0
H_5	0	3	0	7	0	0	6	4
H_6	4	0	0	0	0	0	3	0
H_7	0	0	0	0	6	3	0	2
H_8	7	0	0	0	4	0	2	0

The tour starts from area H_1 and then select the minimum cost area reachable from H_1.

	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8
(1) H_1	0	5	0	6	0	(4)	0	7
H_2	5	0	2	4	3	0	0	0
H_3	0	2	0	1	0	0	0	0
H_4	6	4	1	0	7	0	0	0
H_5	0	3	0	7	0	0	6	4
H_6	4	0	0	0	0	0	3	0
H_7	0	0	0	0	6	3	0	2
H_8	7	0	0	0	4	0	2	0

Mark area H_6 because it is the minimum cost area reachable from H_1 and then select minimum cost area reachable from H_6.

	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8
(1) H_1	0	5	0	6	0	(4)	0	7
H_2	5	0	2	4	3	0	0	0
H_3	0	2	0	1	0	0	0	0
H_4	6	4	1	0	7	0	0	0
H_5	0	3	0	7	0	0	6	4
(2) H_6	4	0	0	0	0	0	(3)	0
H_7	0	0	0	0	6	3	0	2
H_8	7	0	0	0	4	0	2	0

Mark area H_7 because it is the minimum cost area reachable from H_6 and then select minimum cost area reachable from H_7.

	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8
(H1)	0	5	0	6	0	(4)	0	7
H2	5	0	2	4	3	0	0	0
H3	0	2	0	1	0	0	0	0
H4	6	4	1	0	7	0	0	0
H5	0	3	0	7	0	0	6	4
(H6)	4	0	0	0	0	0	(3)	0
(H7)	0	0	0	0	6	3	0	(2)
H8	7	0	0	0	4	0	2	0

Mark area H_8 because it is the minimum cost area reachable from H_8.

	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8
(H1)	0	5	0	6	0	(4)	0	7
H2	5	0	2	4	3	0	0	0
H3	0	2	0	1	0	0	0	0
H4	6	4	1	0	7	0	0	0
H5	0	3	0	7	0	0	6	4
(H6)	4	0	0	0	0	0	(3)	0
(H7)	0	0	0	0	6	3	0	(2)
(H8)	7	0	0	0	(4)	0	2	0

Mark area H_5 because it is the minimum cost area reachable from H_5.

	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8
(H1)	0	5	0	6	0	(4)	0	7
H2	5	0	2	4	3	0	0	0
H3	0	2	0	1	0	0	0	0
H4	6	4	1	0	7	0	0	0
(H5)	0	(3)	0	7	0	0	6	4
(H6)	4	0	0	0	0	0	(3)	0
(H7)	0	0	0	0	6	3	0	(2)
(H8)	7	0	0	0	(4)	0	2	0

Mark area H_2 because it is the minimum cost area reachable from H_2.

	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8
H_1	0	5	0	6	0	(4)	0	7
H_2	5	0	(2)	4	3	0	0	0
H_3	0	2	0	1	0	0	0	0
H_4	6	4	1	0	7	0	0	0
H_5	0	(3)	0	7	0	0	6	4
H_6	4	0	0	0	0	0	(3)	0
H_7	0	0	0	0	6	3	0	(2)
H_8	7	0	0	0	(4)	0	2	0

Mark area H_3 because it is the minimum cost area reachable from H_3.

	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8
H_1	0	5	0	6	0	(4)	0	7
H_2	5	0	(2)	4	3	0	0	0
H_3	0	2	0	(1)	0	0	0	0
H_4	6	4	1	0	7	0	0	0
H_5	0	(3)	0	7	0	0	6	4
H_6	4	0	0	0	0	0	(3)	0
H_7	0	0	0	0	6	3	0	(2)
H_8	7	0	0	0	(4)	0	2	0

Mark area H_4 and then select the minimum cost area reachable from H_4 it is H_1.So, using the greedy strategy, we get the following.

$$H_1 \xrightarrow{4} H_6 \xrightarrow{3} H_7 \xrightarrow{2} H_8 \xrightarrow{4} H_5 \xrightarrow{3} H_2 \xrightarrow{2} H_3 \xrightarrow{1} H_4 \xrightarrow{6} H_1.$$

Thus the minimum travel cost = 4 + 3 + 2 + 4 + 3 + 2 + 1 + 6 = 25

What is Heuristics?

A heuristic is a technique that is used to solve a problem faster than the classic methods. These techniques are used to find the

approximate solution of a problem when classical methods do not. Heuristics are said to be the problem-solving techniques that result in practical and quick solutions.

Heuristics are strategies that are derived from past experience with similar problems. Heuristics use practical methods and shortcuts used to produce the solutions that may or may not be optimal, but those solutions are sufficient in a given limited timeframe.